büyük
yayınları

CEREN'e

YENİ SINAV SİSTEMİNE GÖRE HAZIRLANMIŞTIR

Lise 2 + ÖYS
İÇİN

TRİGONOMETRİ

Mahiye Mangır
2012

- Detaylı konu açıklaması
- Temel teoremler ve ispatlar
- ÖYS öncesi çözülmesi zorunlu problem tipleri

Birkan Dündar

HER HAKKI SAKLIDIR.

Kitabın konu özeti ve testleri hiçbir şekilde alınamaz, teksir edilemez ve muhtelif araçlarla çoğaltılamaz.

BÜYÜK DİZGİ-GRAFİK

Fuat GÖKERKEK Ali AKKAYA

G.M.K. Bulvarı No:16 Tel: 230 47 20 - 230 22 84

BASILDIĞI YER:
Büyük Dersane Matbaası
Tel: 230 21 00 / Ankara

SUNUŞ

Yükseköğretim için yapılan ÖSYS (öğrenci seçme ve yerleştirme sınavı) ki aşamalı bir sınav sistemidir.

ÖSS adı verilen birinci aşama sınavında, adayların, Türkçeyi kullanma e matematiksel ilişkilerden yararlanma gücünü sosyal bilimlerle fen bilimlerindeki temel kavram ve ilkelerle düşünme becerisini ölçen sorulara, ÖYS adı verilen ikinci aşama sınavında ise bu bilim dallarındaki bilgi düzeyini ölçen sorulara ağırlık verilmektedir.

Girmek istediğiniz ve ilk tercihleriniz arasında yer alan fakültelerden birine girebilmeniz için bilgi birikiminizi artırıp yeterli bilgi düzeyine ulaşmanız yanında, sosyal bilimlerle fen bilimlerinin temel kavram ve ilkeleriyle düşünme becerisi kazanmanız, öğrendiğiniz ya da verilen bilgilerden yararlanma ve bunları kullanma alışkanlığı edinmeniz de gereklidir.

ÖSYS için yapacağınız yaygın kullanımı ile üniversiteye hazırlık deneme çalışmalar sırasında, ÖSYS için yeterli bilgi düzeyine ulaşmanıza, bilimsel düşünme becerisi kazanmanıza, çeşitli bilgilerden yararlanma ve bunları kullanma alışkanlığı edinmenize yardımcı olacak şekilde hazırlanmış kitaplara ihtiyacınız olacaktır.

Kitap seçiminin sınav hazırlığını, sınav hazırlığının da fakülte seçimindeki tercihleri etkileyebileceği unutulmamalı ve kitap seçimine önem verilmelidir.

Büyük Yayınları ÖSYS'ye hazırlık ihtiyaçlarınıza cevap verebilecek güvenilir yayınlardır.

"Dersaneyi eve götürme özelliği" Büyük Yayınları 'nın önemli bir özelliğidir ve bu özellik Büyük Yayınları'nı benzerlerinden ayıran özelliklerin başında gelir.

Başarı dileklerimle.

Hikmet DOĞAY

BÜYÜK YAYINLARI KURUCUSU

ÖNSÖZ

Değerli Öğrenciler,

Lise 2 öğrencilerinin ve ÖYS hazırlığı yapan öğrencilerin kaynak sıkıntısı çektikleri bir kitap daha elinize geçiyor.

Geometri kitabının gördüğü büyük ilgi ve 1 sene dolmadan gerçekleşen 3. baskısı, meslektaşlarımın ve öğrencilerimin ısrarı üzerine trigonometri ders notlarımı da yayınlıyorum. Bu defa elinizde bol çözüm ve ispat var. Bölüm sonundaki testler de bilgi eksikliğinizi ve kıyaslama gücünüzü ölçecek şekilde düzenlenmiştir. Kitabı kullanırken özellikle seçilmiş örnekleri ve çözümlerini okuduktan sonra bir kerede siz yapmaya çalışınız. Başarınızdaki katkıyı görmekten büyük mutluluk duyacağım.

Kitabın yayınlanması için hiçbir fedakârlıktan kaçınmayan Büyük Dersane Genel Müdürü Sayın C. Rumi Doğay'a teşekkür ederim.

Kitap için (+), (–) tüm görüşler için şimdiden teşekkür ederim. Aynı konuda çıkacak diğer kitapları şimdiden selamlıyorum.

Birkan DÜNDAR
Ankara - Mart - 1988

TEŞEKKÜR

Kitabın redaksiyonu için yardımlarını esirgemeyen Sayın Nimet Dündar, Nurkan Dündar, Tutku Özer'e ve testleri çözen öğrencilerim Kenan Sönmez ve Erkin Urkan'a ve mızıkçılık yapmayan kızım Ceren Dündar'a, kitabın yazımını yapan sabırlı IBM operatörü Fuat Gökerkek ve şekil - sahife dizaynını yapan Ali Akkaya'ya ve Büyük Dersane Matbaası çalışanlarına (+ ∞) teşekkür ederim.

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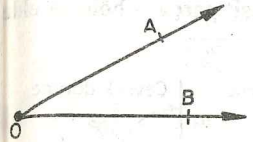
1. BÖLÜM

AÇILAR, AÇI ÖLÇÜLERİ ve BİRİM ÇEMBER

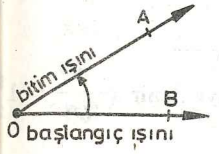
AÇI :



Uç noktaları ortak iki ışının birleşimine açı denir.

[OA ∪ [OB = $\widehat{AOB}$ = $\widehat{BOA}$ = $\hat{O}$ dir.

YÖNLÜ AÇI :



(+) **Pozitif açı**

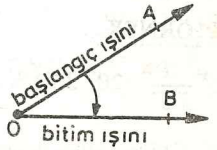
(Saat yönünün tersi)

$\widehat{BOA}$ açısı olarak okunur.

(−) **Negatif açı**

(Saat yönünde)

$\widehat{AOB}$ açısı olarak okunur.

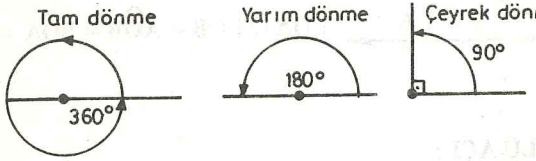


ÇI ÖLÇÜLERİ :

çı ve yay birimleri; 1) Derece 2) Radyan 3) Grad

1) **DERECE** Bir yılın 360 gün olduğunu düşünen Babilli ma tikçiler, dünyanın güneş etrafında dönmesi fikrinden hareke rek daireyi 360 eşit parçaya bölmüş ve her eşit parçaya 1 ((1°) demişlerdir. Böylece yarım dönme 180° ve çeyrek dönm olarak tanımlanmıştır.

Bizde, 1 derecelik yayı; çemberi 360 eşit parçaya bölerek eld ceğiz.



a) Dakika : Bir derecenin $\frac{1}{60}$ 'ına 1 dakika denir. ($\frac{1^\circ}{60} = 1$

1 dakika 1^D veya 1' olarak gösterilir.

b) Saniye : Bir dakikanın $\frac{1}{60}$ 'ına 1 saniye denir. ($\frac{1^D}{60} = 1$

1 saniye 1^S veya 1'' olarak gösterilir.

5 derece 19 dakika 23 saniyelik açı 5° 19' 23'' şeklinde ya

ÖRNEK :

a) $\frac{1}{5}(20^\circ\ 35') = 4^\circ\ 7'$

b) $\frac{1}{2}(125^\circ\ 16') = \frac{1}{2}(124^\circ\ 76') = 62^\circ\ 38'$

c) $\frac{1}{2}(33^\circ\ 17') = \frac{1}{2}(32^\circ\ 77') = \frac{1}{2}(32^\circ\ 76'\ 60'') = 16^\circ\ 38$

UYARI : 1° = 60' ve 1' = 60'' olarak dönüşüm yapınız.

ÖRNEK :

a) 23° 30' 19'' ve 35° 23' 10'' açılarını toplayınız.

Derece	Dakika	Saniye
23°	30'	19''
+ 35°	23'	10''
58°	53'	29''

b) 43° 27' 13'' ve 18° 39' 52'' açılarını toplayınız.

43°	27'	13''	
+ 18°	39'	52''	
61°	66'	65''	[60'ın katlarını yer değiştirelim]
61°	67'	5''	[60'ın katlarını yer değiştirelim]
SONUÇ : 62°	7'	5''	

c) A = 17° 13' 18'' ve B = 9° 23' 38'' ise, A – B nedir ?

$$\begin{array}{r} 17^\circ\ 13'\ 18'' \\ -\ 9^\circ\ 23'\ 38'' \\ \hline \end{array} \Rightarrow \begin{array}{r} 17^\circ\ \overset{+60''}{12'}\ 78'' \\ -\ 9^\circ\ 23'\ 38'' \\ \hline \end{array} \Rightarrow \begin{array}{r} 16^\circ\ \overset{+60'}{72'}\ 78'' \\ -\ 9^\circ\ 23'\ 38'' \\ \hline \end{array}$$

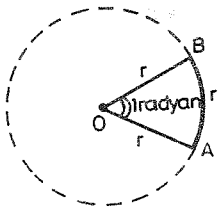
SONUÇ $\boxed{7^\circ\ 49'\ 40''}$ dir.

) 3782'' nin derece, dakika, saniye cinsinden eşiti nedir ?

$$\begin{array}{r|l} 3782 & 60 \\ \cline{2-2} -\ 360 & 63 \\ \hline 182 & \\ -\ 180 & \\ \hline 2 & \end{array}$$

3782'' = 63' 2'' = 1° 3' 2''

2) **RADYAN** Bir çemberin yarıçapı uzunluğundaki yaya 1 Ra yanlık yay denir.

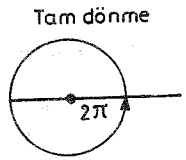


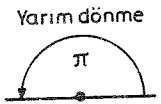
* Çemberde bir dönmenin kaç radya olduğunu bulmak için $\frac{\text{çevre}}{\text{yarıçap}}$ oran kullanılır.

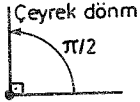
çevre $= 2\pi r$
yarıçap $= r$
} iken

1 devirde, $\frac{2\pi r}{r} = 2\pi$ radyan vardı

SONUÇ :	1 devir ,	2π radyan = 360 derece
	Yarım devir ,	π radyan = 180 derece
	Çeyrek devir ,	$\pi/2$ radyan = 90 derece





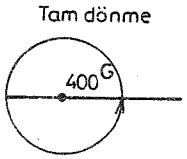


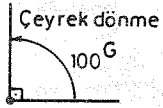
UYARI : Açıları radyan cinsinden kullanırken, açının içinde π var π radyan yerine sadece π diyeceğiz.

3) **GRAD** Çemberin 400'e bölünmesi ile bir grad'lik yay elde e lir. 1 grad, 1^G olarak gösterilir.

a) grad dakikası $= \frac{1}{100} \cdot 1^G$ $\quad (1^C = \frac{1^G}{100})$

b) grad saniyesi $= \frac{1}{100} \cdot 1^C$ $\quad (1^{CC} = \frac{1^C}{100})$





SONUÇ :

Tam devir : 360 derece = 2π radyan = 400 grad

Yarım devir : 180 derece = π radyan = 200 grad

Çeyrek devir : 90 derece = $\frac{\pi}{2}$ radyan = 100 grad

AÇI ÖLÇÜLERİNİN BİRBİRLERİNE DÖNÜŞTÜRÜLMESİ

Yukarıda elde ettiğimiz sonuca göre;

A) $\frac{\text{Derece}}{360} = \frac{\text{Radyan}}{2\pi} = \frac{\text{Grad}}{400}$

B) $\frac{\text{Derece}}{180} = \frac{\text{Radyan}}{\pi} = \frac{\text{Grad}}{200}$

C) $\frac{\text{Derece}}{90} = \frac{\text{Radyan}}{\pi/2} = \frac{\text{Grad}}{100}$ dönüşümlerinden biri kullanılabilir.

ÖRNEK :

1) $\frac{\pi}{4}$ radyanlık açının derece cinsinden ölçüsü nedir ?

UYARI : Radyan cinsinden verilen açıyı derece cinsinden yazmak için π yerine 180 yazınız.

$$\frac{\pi}{4} \text{ radyan} = \frac{180}{4} \text{ derece} = 45^\circ$$

II. Yol : $\frac{\text{Derece}}{360} = \frac{\text{Radyan}}{2\pi}$ den,

$$\frac{\text{Derece}}{360} = \frac{\pi/4}{2\pi} \Rightarrow \text{Derece} = 360 \cdot \frac{\pi}{4} \cdot \frac{1}{2\pi} = 45$$

2) 60 derecelik açının radyan cinsinden ölçüsü nedir ?

$$\boxed{\frac{D}{360} = \frac{R}{2\pi}} \Rightarrow \frac{60}{360} = \frac{R}{2\pi} \Rightarrow R = \frac{60 \cdot 2\pi}{360} = \frac{\pi}{3}$$

II. Yol : 180'i kaça bölersek 60 elde ederiz ?

$$\frac{180}{3} = 60 \Rightarrow \frac{\pi}{3} \text{ radyan} = 60^\circ$$

3) 45 derecelik açının grad cinsinden ölçüsü nedir ?

$$\boxed{\frac{D}{180} = \frac{G}{200}} \Rightarrow \frac{45}{180} = \frac{G}{200} \Rightarrow G = \frac{45 \cdot 200}{180} = 50$$

$$\Rightarrow 45^\circ = 50^G$$

UYARI : Trigonometride en çok kullanılan açılar 30°, 45°, 6[0°] ve bu açıların katlarıdır. Bu açıların katlarının radyan cinsine çev[irmesini] iyi öğrenmelisiniz.

[0° ve 60° için,]

$30^\circ = \frac{\pi}{6}$ radyan

$60^\circ = \frac{\pi}{3}$ radyan [2 . 30°]

$90^\circ = \frac{\pi}{2}$ radyan [3 . 30°]

$120^\circ = 2 \cdot \frac{\pi}{3}$ " [2 . 60°]

$150^\circ = \frac{5\pi}{6}$ radyan [5.30°]

$180^\circ = \pi$ radyan [6.30°]

$210^\circ = \frac{7\pi}{6}$ radyan [7 . 30°]

$240^\circ = \frac{4\pi}{3}$ radyan [4 . 60°]

$300^\circ = \frac{5\pi}{3}$ radyan [5 . 60°]

$360^\circ = 2\pi$ radyan [6 . 60°]

b) **[45° için,]**

$45^\circ = \frac{\pi}{4}$ radyan

$90^\circ = \frac{\pi}{2}$ radyan (2.45°)

$135^\circ = \frac{3\pi}{4}$ radyan (3.45°)

$180^\circ = \pi$ radyan (4.45°)

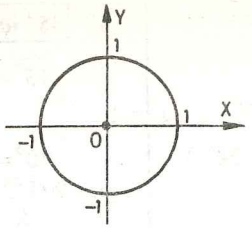
$225^\circ = \frac{5\pi}{4}$ radyan (5.45°)

$270^\circ = \frac{3\pi}{2}$ radyan (6.45°)

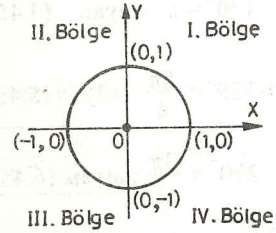
$315^\circ = \frac{7\pi}{4}$ radyan (7.45°)

$360^\circ = 2\pi$ radyan (8.45°)

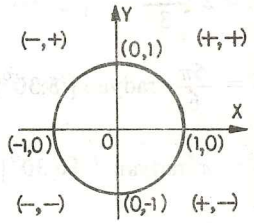
İM ÇEMBER : Merkezi, eksenlerin kesim noktası ve yarıçapı 1 [birim] uzunlukta olan yönlü çembere birim çember ya da trigonometrik çember denir. Denklemi $x^2 + y^2 = 1$ dir. [M (0, 0), r = 1]



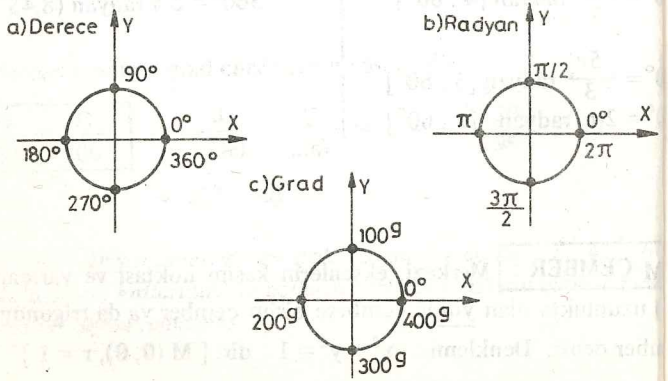
* **Birim çemberde bölgeler**



* **Bölgelerin işaretleri**

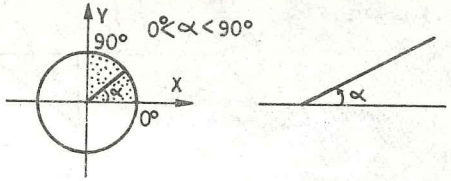


* **Birim çemberde açı ölçüleri**

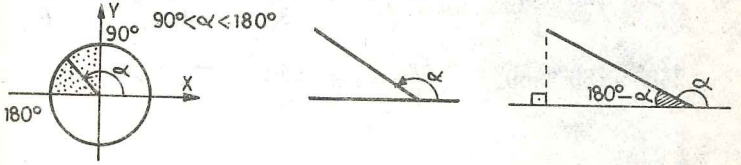


* **Birim çemberde bölgelerin açı yazılımları**

I. Bölge :



II. Bölge :



II. Bölgede açılar $180° - \alpha$, $(\pi - \alpha)$ şeklinde düzenlenir.

ÖRNEK :

$120° = 180° - 60°$

$150° = 180° - 30°$

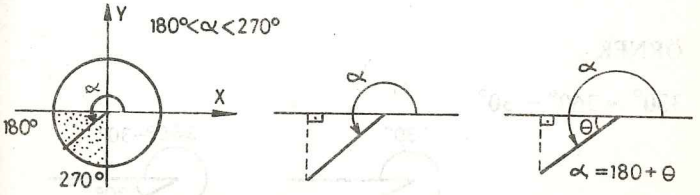
$135° = 180° - 45°$

$\frac{2\pi}{3} = \pi - \frac{\pi}{3}$

$\frac{3\pi}{4} = \pi - \frac{\pi}{4}$

$\frac{5\pi}{6} = \pi - \frac{\pi}{6}$

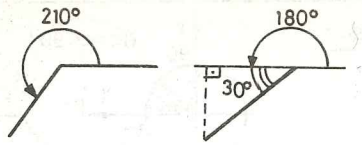
III. Bölge :



III. Bölgede açılar $180 + \alpha$; $(\pi + \alpha)$ şeklinde düzenlenir.

ÖRNEK :

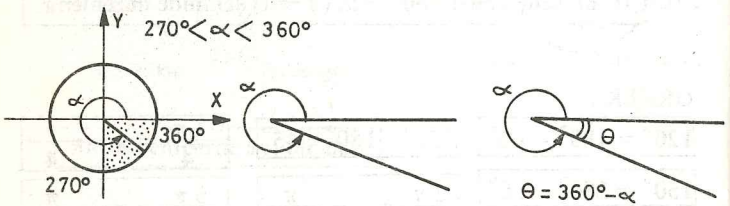
1) $210° = 180° + 30°$



2) $225° = 180° + 45°$, $\frac{4\pi}{3} = \pi + \frac{\pi}{3}$, $\frac{7\pi}{6} = \pi + \frac{\pi}{6}$

$240° = 180° + 60°$, $\frac{5\pi}{4} = \pi + \frac{\pi}{4}$

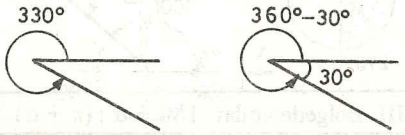
IV. Bölge :



IV. Bölgede açılar $360 - \alpha$; $(2\pi - \alpha)$ şeklinde düzenlenir.

ÖRNEK :

1) $330° = 360° - 30°$

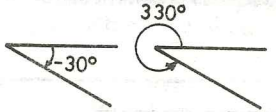


) $315° = 360° - 45°$ $\quad \frac{5\pi}{3} = 2\pi - \frac{\pi}{3}$

$300° = 360° - 60°$ $\quad \frac{7\pi}{4} = 2\pi - \frac{\pi}{4}$

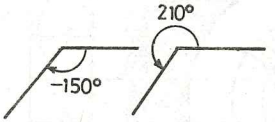
NEGATİF AÇILAR :

) $-30°$



$\Rightarrow -30° \equiv 330°$

) $-150°$



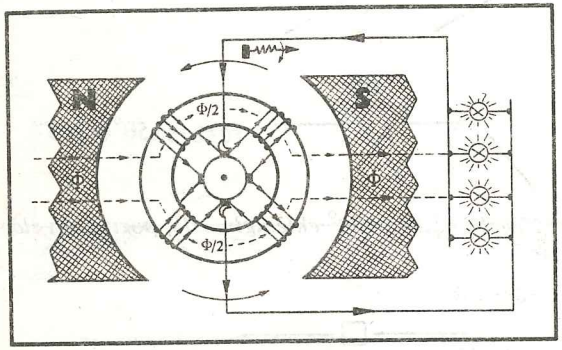
$\Rightarrow -150° \equiv 210°$

UYARI : Negatif açılara $360°$ eklediğimizde, pozitif açı elde ederiz.

büyük
yayınları

YENİ
PROGRAMA UYGUN

Lise 2 FİZİK



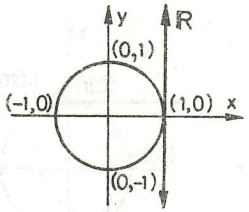
• Konu Açıklamalı
• Örnek Çözümlü
• Cevaplı Sorular

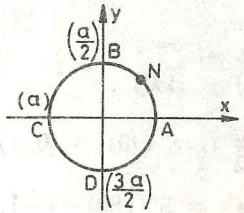
NURETTİN CERAN • NECATİ TANRIÖVER

2. BÖLÜM

SARMAL (DAİRESEL) FONKSİYON

Reel sayılar kümesinin elemanları ile birim çemberin üzerindeki noktalar arasında bir eşleme kurulabilir. $a \in R^+$ olmak üzere [0,2 a) aralığındaki sayılar ile birim çemberin noktalarını bire - bir eşleyebiliriz.





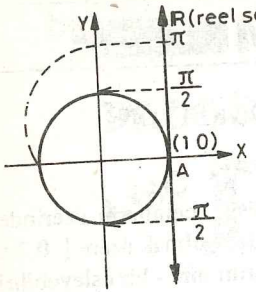
* Bu eşlemede A, sıfıra; B, $\frac{a}{2}$ 'ye C, a'ya D, $\frac{3a}{2}$ 'ye karşılık gelir.

O halde çember üzerindeki herhangi bir N noktasına, [0, 2a) aralığından bir sayı eşlenecektir. Eşlenen bu sayıya α dersek; $\alpha + 2a \,.\, k$ sayılarında N noktasına eşlenecektir. (Neden ?) $(k \in Z)$

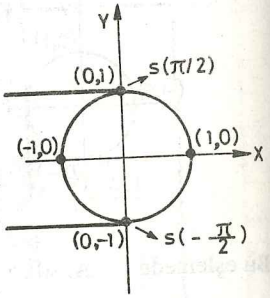
Buradan $\theta = \alpha + 2a \,.\, k$ eşitliğinde α ; θ nın esas ölçüsü olur. (Esas ölçüyü pozitif yapacak şekilde $k \in Z$ bulmalıyız.)

Artık yukarıda tanımladığımız eşlemeyi $S : R \to Ç$ şeklinde gösterebiliriz.

Reel sayıları çemberin üzerindeki noktalara eşleyen S (sarmal) fonksiyonu]



Analitik düzlemde $\{(1,t) \mid t \in$ kümesi birim çembere (1, 0) da ğet olan bir sayı eksenidir. S fo siyonu 0 sayısını A (1, 0) a eşles Böylece gerçel sayı ekseninin noktası birim çemberin bir ve nız bir noktası ile çakışır.



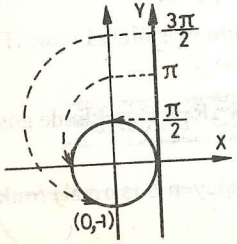
$S(0) = (1,0)$

$S(\frac{\pi}{2}) = S'(90) = (0,1)$

$S(\pi) = S'(180) = (-1,0)$

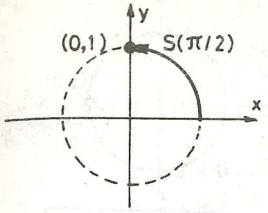
$S(\frac{3\pi}{2}) = S'(270) = (0,-1)$ dir.

ÖRNEK : $S(\frac{3\pi}{2}) = (0,-1)$

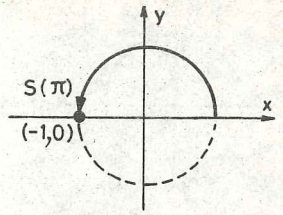


Reel sayıları ip, birim çemberi makara olarak düşünürsek, $\frac{3\pi}{2}$ nin (0, – 1) e eşleneceği daha açık görülür.

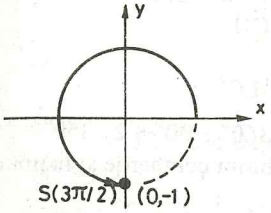
Aşağıdakileri dikkatle inceleyiniz.
(S radyan, S' derece cinsinden)



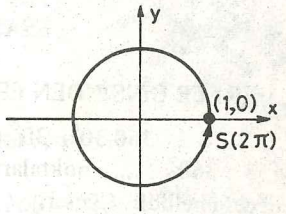
$S(\frac{\pi}{2}) = (0,1)$



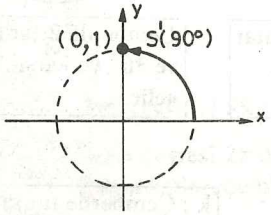
$S(\pi) = (-1,0)$



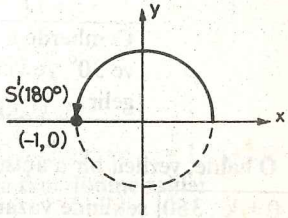
$S(\frac{3\pi}{2}) = (0,-1)$



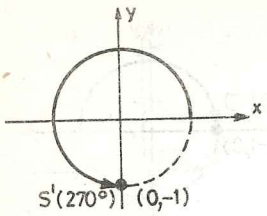
$S(2\pi) = (1,0)$



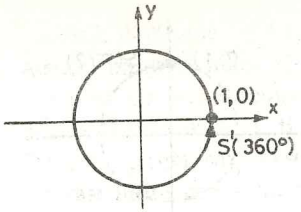
$S'(90) = (0,1)$



$S'(180) = (-1,0)$



S' (270) = (0, – 1)

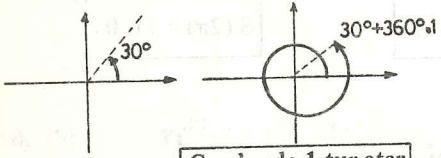


S' (360) = (1, 0)

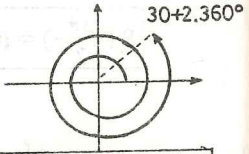
ESAS ÖLÇÜ

1) DERECE CİNSİNDEN ESAS ÖLÇÜ :

... 30 – 1 . 360,30°, 30°+ 1 . 360° , 30°+ 2 . 360° , 30°+ 3 . 360° , ... noktalarının birim çemberde aynı noktaya eşleneceği bellidir. Çünkü;



Çemberde 1 tur atar ve 30° ye karşılık gelir.



Çemberde 2 tur atar ve 30° ye karşılık gelir.

O halde, verilen bir α açısını,

$\alpha = \theta + k \cdot 360$ şeklinde yazarsak $(0° \leq \theta < 360°)$

k : Çemberde tur sayısı
θ : Esas ölçü olur.

[Kısaca mod (360) denebilir.]

ÖRNEK : 2240° lik açının esas ölçüsü nedir ?

$2240° = \theta + k \cdot 360°$ ise, θ = ?
k = ?

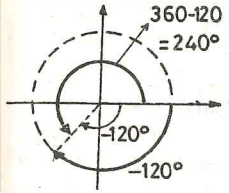
2240° lik açının birim çemberde kaç tur döndüğünü bulmak için 360'a böleceğiz.

2 2 4 0	360
– 2 1 6 0	6 → (tur sayısı)
0 0 8 0	→ esas ölçü

O halde 2240° lik açının esas ölçüsü 80° dir.

$2240 = 6 \cdot 360 + \boxed{80°}$

ÖRNEK : – 120° nin esas ölçüsü nedir ?



Negatif açılara 360° ekleyerek esas ölçü bulunur.

360° – 120° = 240° dir.

2) RADYAN CİNSİNDEN ESAS ÖLÇÜ :

Birim çemberin çevresi 2π dir. ($Ç = 2\pi r$ ve $r = 1$). O halde 2π ve 2π nin tam katları çemberde tam dönme sağlar.

1. Tur 2π
2. Tur $2 . (2\pi)$
3. Tur $3 . (2\pi)$
4. Tur $4 . (2\pi)$

k . tur $k(2\pi)$ dir.

O halde

$\theta, \theta + 2\pi, \theta + 2(2\pi), \theta + 3(2\pi), \ldots, \theta + k(2\pi)$ açıla çemberde aynı noktaya eşleneceklerdir.

Verilen α açısını,

$\boxed{\alpha = \theta + k \cdot 2\pi}$ şeklinde yazarsak k : Çemberde tur sayısı
θ : Esas ölçü

ÖRNEK :

a) $19\pi = \textcircled{\pi} + \textcircled{9}(2\pi) \Rightarrow$ Çemberde 9 tur atar ve esas ölçüsü olu
esas ölçü Tur sayısı

b) $24\pi = 12(2\pi) \Rightarrow$ Çemberde 12 tam tur atar ve esas ölçü 0 dır.

c) $\frac{26\pi}{3} = \frac{A}{3} + \frac{B}{3}$ şeklinde yazılır.

(A = Paydanın çift katı ve 26π ye yakın)

$3 \cdot \underbrace{(8\pi)}_{\text{çift kat}} = 24\pi = A$

$= \frac{24\pi}{3} + \frac{2\pi}{3} = 8\pi + \frac{2\pi}{3} = \underbrace{4(2\pi)}_{\substack{\text{çemberde} \\ \text{4 tur atar}}} + \boxed{\frac{2\pi}{3}}$ → esa ölç

d) $\frac{47\pi}{4} = \frac{40\pi}{4} + \frac{7\pi}{4} = 10\pi + \frac{7\pi}{4} = \underbrace{5(2\pi)}_{\substack{\text{çemberde} \\ \text{5 tur atar}}} + \boxed{\frac{7\pi}{4}}$ esas öl

e) $\frac{-37\pi}{4} = \frac{-40\pi}{4} + \frac{3\pi}{4} = -10\pi + \frac{3\pi}{4}$

$= \underbrace{-5(2\pi)}_{\substack{\text{çemberde} \\ \text{negatif yönde 4 tur atar}}} + \frac{3\pi}{4}$

f) $\frac{33\pi}{5}$ in esas ölçüsü nedir ?

Verilen açıyı 2π ye böleriz.

$\frac{33\pi}{5} \;\Big|\; 2\pi = \frac{10\pi}{5}$

$\frac{30\pi}{5} \;\Big|\; 3$

$= \frac{3\pi}{5}$

$\frac{33\pi}{5} = \underbrace{3 \cdot (2\pi)}_{\substack{\text{çemberde} \\ \text{3 tur atar}}} + \boxed{\frac{3\pi}{5}}$ → esas ölçü

ALIŞTIRMALAR

1. Aşağıdakilerin derece cinsinden esas ölçülerini bulunuz.

A) $1780°$ B) $1920°$ C) $2587°$ D) $-1360°$ E) $-4310°$

2. Aşağıdakilerin radyan cinsinden esas ölçülerini bulunuz.

A) $\frac{49\pi}{3}$ B) $\frac{413\pi}{5}$ C) $-\frac{57\pi}{6}$

D) $-\frac{123\pi}{4}$ E) $-\frac{139\pi}{4}$

ALIŞTIRMALARIN ÇÖZÜMÜ

a) $1780^\circ \mid 360^\circ$
$1440 \mid 4 \rightarrow$ tur sayısı
340°
esas ölçü

b) $1920^\circ \mid 360^\circ$
$1800 \mid 5 \rightarrow$ tur sayısı
120
esas ölçü

c) $2587^\circ \mid 360^\circ$
$2520 \mid 7$
67°
esas ölçü

d) $-1360^\circ \mid 360^\circ$
$\pm 1440 \mid -4$
80°
esas ölçü

e) $4310^\circ \mid 360^\circ$
$360 \mid 11$
710
360
350

$-(4310^\circ) \equiv -(350^\circ) \equiv -350^\circ + 360^\circ \equiv \boxed{10^\circ}$

2) a) $\frac{49\pi}{3} \mid 2\pi = \frac{6\pi}{3}$
$\frac{48\pi}{3} \mid 8 \rightarrow$ tur sayısı
$\frac{\pi}{3}$ → esas ölçü

b) $\frac{413\pi}{5} \mid 2\pi = \frac{10\pi}{5}$
$\frac{410\pi}{5} \mid 41 \rightarrow$ tur sayısı
$\frac{3\pi}{5}$ → esas ölçü

c) $\frac{-57\pi}{6} \mid 2\pi = \frac{12\pi}{6}$
$\frac{\pm 60\pi}{6} \mid -5$ (negatif yönde 4 tur)
$\frac{3\pi}{6}$
esas ölçü

d) $\frac{-123\pi}{4} \mid 2\pi = \frac{8\pi}{4}$
$\frac{\pm 128\pi}{4} \mid -16$
$\frac{5\pi}{4}$
esas ölçü

e) $\frac{-139\pi}{4} = \frac{-144\pi}{4} + \frac{5\pi}{4} = -36\pi + \frac{5\pi}{4}$

$= \underbrace{-18(2\pi)}_{2\pi \text{ in katlarını atarız}} + \boxed{\frac{5\pi}{4}}$ → esas ölçü

ALIŞTIRMALAR

1. Aşağıdaki açıların kendi türünde esas ölçüsünü bulunuz.

A) -75° B) 1803° C) 1483°
D) -1347° E) -2000° F) $\frac{28\pi}{3}$
G) $\frac{36\pi}{5}$ H) $\frac{-47\pi}{4}$ İ) $\frac{-57\pi}{6}$
J) $\frac{-211\pi}{4}$

Yanıtlar :
a) 285° B) 3° C) 43° D) 93°

E) 160° F) $\frac{4\pi}{3}$ G) $\frac{6\pi}{5}$ H) $\frac{\pi}{4}$

İ) $\frac{\pi}{2}$ J) $\frac{5\pi}{4}$

2. Aşağıdaki açıların esas ölçüsü bulunurken birim çemberde kaç tam dönüş yaptığını bulunuz.

A) 370° B) 780° C) 1310° D) 1520°

E) 1988° F) $\frac{13\pi}{4}$ G) $\frac{28\pi}{3}$ H) $\frac{37\pi}{5}$

İ) $\frac{421\pi}{6}$ J) $\frac{247\pi}{7}$

Yanıtlar :

A) 1 B) 2 C) 3 $\left(\begin{array}{l|l} 1310 & 360 \\ -1080 & 3 \rightarrow \text{tur sayısı} \\ \hline 230 & \\ \downarrow & \\ \text{esas ölçü} & \end{array}\right)$ D) 4

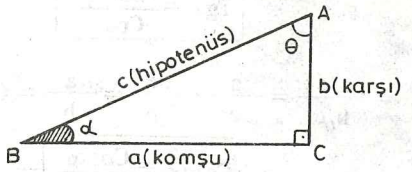
E) 5 F) 1 G) 4 H) 3 $\left[\begin{array}{l|l} \frac{37\pi}{5} & 2\pi = \frac{10\pi}{5} \\ \hline \frac{30\pi}{5} & \\ = \frac{7\pi}{5} & \text{tur sayısı} \end{array}\right]$

İ) 35 J) 17

3. BÖLÜM

DİK ÜÇGENDE TRİGONOMETRİK ORANLAR

TANIM :



$$\text{Sinüs } (\alpha) = \text{Sin } \alpha = \frac{\text{Karşı dik kenar}}{\text{Hipotenüs}} = \frac{b}{c}$$

$$\text{Kosinüs } (\alpha) = \text{Cos } \alpha = \frac{\text{Komşu dik kenar}}{\text{Hipotenüs}} = \frac{a}{c}$$

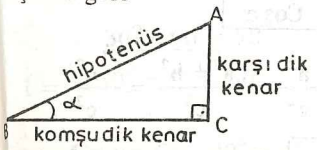
$$\text{Tanjant } (\alpha) = \text{Tan } \alpha = \text{Tg } \alpha = \frac{\text{Karşı dik kenar}}{\text{Komşu dik kenar}} = \frac{b}{a}$$

$$\text{Kotanjant } (\alpha) = \text{Cotg } \alpha = \frac{\text{Komşu dik kenar}}{\text{Karşı dik kenar}} = \frac{a}{b}$$

$$\text{Sekant } (\alpha) = \text{Sec } \alpha = \frac{\text{Hipotenüs}}{\text{Komşu dik kenar}} = \frac{c}{a}$$

$$\text{Kosekant } (\alpha) = \text{Cosec } \alpha = \frac{\text{Hipotenüs}}{\text{Karşı dik kenar}} = \frac{c}{b}$$

α açısına göre



θ açısına göre

Kullandığımız dar açının yerine göre, dik kenarların karşı ve komşu olarak değişebileceğine dikkat ediniz.

SONUÇLAR :

① $\dfrac{\text{Sin}\,\alpha}{\text{Cos}\,\alpha} = \dfrac{b/c}{a/c} = \dfrac{b}{c} \cdot \dfrac{c}{a} = \dfrac{b}{a} = \text{Tg}\,\alpha$

$$\Rightarrow \boxed{\text{Tg}\,\alpha = \frac{\text{Sin}\,\alpha}{\text{Cos}\,\alpha}}$$

② $\dfrac{\text{Cos}\,\alpha}{\text{Sin}\,\alpha} = \dfrac{a/c}{b/c} = \dfrac{a}{c} \cdot \dfrac{c}{b} = \dfrac{a}{b} = \text{Cotg}\,\alpha$

$$\Rightarrow \boxed{\text{Cotg}\,\alpha = \frac{\text{Cos}\,\alpha}{\text{Sin}\,\alpha}}$$

③ $\text{Tg}\,\alpha \,.\, \text{Cotg}\,\alpha = \dfrac{b}{a} \cdot \dfrac{a}{b} = 1 \Rightarrow \boxed{\text{Tg}\,\alpha \,.\, \text{Cotg}\,\alpha = 1}$

$$\left[\ \frac{\text{Sin}\,\alpha}{\text{Cos}\,\alpha} \cdot \frac{\text{Cos}\,\alpha}{\text{Sin}\,\alpha} = 1\ \right]$$

④ $\text{Sin}\,\alpha \,.\, \text{Cosec}\,\alpha = \dfrac{b}{c} \cdot \dfrac{c}{b} = 1 \Rightarrow \text{Sin}\,\alpha \,.\, \text{Cosec}\,\alpha = 1$

$$\Rightarrow \boxed{\text{Cosec}\,\alpha = \frac{1}{\text{Sin}\,\alpha}}$$

⑤ $\text{Cos}\,\alpha \,.\, \text{Sec}\,\alpha = \dfrac{a}{c} \cdot \dfrac{c}{a} = 1 \Rightarrow \text{Cos}\,\alpha \,.\, \text{Sec}\,\alpha = 1$

$$\Rightarrow \boxed{\text{Sec}\,\alpha = \frac{1}{\text{Cos}\,\alpha}}$$

⑥ $\text{Sin}^2\,\alpha + \text{Cos}^2\,\alpha = \dfrac{b^2}{c^2} + \dfrac{a^2}{c^2} = \dfrac{a^2+b^2}{c^2} = \dfrac{c^2}{c^2} = 1$

$$\Rightarrow \boxed{\text{Sin}^2\,\alpha + \text{Cos}^2\,\alpha = 1}$$

(ABC dik üçgeninde $b^2 + a^2 = c^2$ dir. (Neden ?)

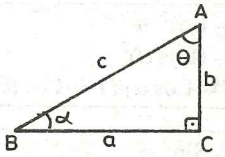
DİKKAT : $(\text{Sin}\,\alpha)^2 = \text{Sin}^2\,\alpha$ ve $(\text{Sin}\,\alpha)^2 \neq \text{Sin}^2\,\alpha^2$.

$[\ \text{Sin}^2\,10 = (\text{Sin}10)^2 = (\text{Sin}10)\,.\,(\text{Sin}10)\ ]$

$$\boxed{\text{Sin}^2\,\alpha \neq \text{Sin}\,\alpha^2}$$

⑦

$\text{Sin}\alpha = \dfrac{b}{c}$, $\text{Cos}\,\theta = \dfrac{b}{c}$	$\text{Tan}\alpha = \dfrac{b}{a}$, $\text{Cotg}\,\theta = \dfrac{b}{a}$
$\text{Cos}\alpha = \dfrac{a}{c}$, $\text{Sin}\,\theta = \dfrac{a}{c}$	$\text{Cotg}\alpha = \dfrac{a}{b}$, $\text{Tan}\,\theta = \dfrac{a}{b}$



$\alpha + \theta = 90°$

(Tümler açılar)

O halde birbirini 90° ye tamamlayan açılardan birinin sinüsü diğerinin kosinüsüne, birinin tanjantı diğerinin kotanjantına ve birinin sekantı diğerinin kosekantına eşittir.

Aşağıdakileri dikkatle inceleyiniz.

1) $\underbrace{20° + 70° = 90°}$ $\quad \text{Sin}20° = \text{Cos}70°$

2) $\underbrace{10° + 80° = 90°}$ $\quad \text{Cos}10° = \text{Sin}80°$

3) $\underbrace{30° + 60° = 90°}$ $\quad \text{tg}\,30° = \text{Cotg}\,60°$

4) $\underbrace{100° - 10° = 90°}$ $\quad \text{Sec}(100) = \text{Cosec}\,(-10)$

5) $\text{Sin}\,\alpha = \underbrace{\text{Cos}\,(90° - \alpha)}$ $\quad \alpha + (90° - \alpha) = 90°$

6) $\text{Sin}(90 + \alpha) = \underbrace{\text{Cos}\,(-\alpha)}$ $\quad (90 + \alpha) + (-\alpha) = 90°$

7) $\text{Tg}\,(70 + 4\alpha) = \text{Cotg}\,(20 - 4\alpha)$

$(70 + 4\alpha) + (20 - 4\alpha) = 90°$

8) $\text{Sin}\,(10 + \frac{7\alpha}{2}) = \text{Cos}\,(80 - \frac{7\alpha}{2})$

$(10 + \frac{7\alpha}{2}) + (80 - \frac{7\alpha}{2}) = 90°$

UYARI : $\text{Sin}^2\,\alpha + \text{Cos}^2\alpha = 1$ özdeşliği sadeleştirmelerde çok sık kullanılır. O halde ;

$$\text{Sin}^2\,\alpha = 1 - \text{Cos}^2\,\alpha = 1 - (\text{Cos}\,\alpha)^2 = (1 - \text{Cos}\,\alpha)(1 + \text{Cos}\,\alpha)$$

ve

$$\text{Cos}^2\,\alpha = 1 - \text{Sin}^2\,\alpha = 1 - (\text{Sin}\,\alpha)^2 = (1 - \text{Sin}\,\alpha)(1 + \text{Sin}\,\alpha)$$

olduğunu sizde kolayca bulabilirsiniz.

NOT: $(a^2 - b^2) = (a - b)(a + b)$ olduğunu hatırladınız mı ?

ALIŞTIRMALAR

1. Aşağıdaki trigonometrik ifadeleri sadeleştiriniz

A) $\tan x \,.\, \text{Cos}\,x$ B) $\text{Sin}\,x \,.\, \text{Cotg}\,x$

C) $\tan\theta \,.\, \text{Cotg}^2\,\theta$ D) $\sqrt{1 - \text{Cos}\,\theta} \cdot \sqrt{1 + \text{Cos}\,\theta}$

E) $\dfrac{\text{Sin}^2\,x - 1}{\text{Cos}^2\,x}$ F) $\text{Sin}^4\,x + \text{Sin}^2\,x \,.\, \text{Cos}^2\,x$

G) $\dfrac{\text{Sin}\,\theta}{\text{Cos}\,\theta} + \dfrac{\text{Cos}\,\theta}{\text{Sin}\,\theta}$ H) $\tan x \,.\, \text{Cotg}\,x$

ÇÖZÜM :

A) $\tan x \,.\, \text{Cos}\,x = \dfrac{\text{Sin}\,x}{\text{Cos}\,x} \cdot \text{Cos}\,x$

$= \boxed{\text{Sin}\,x}$ [$\tan x = \dfrac{\text{Sin}\,x}{\text{Cos}\,x}$ aldık. (Sonuç 1 den)]

B) $\text{Sin}\,x \,.\, \text{Cotg}\,x = \text{Sin}\,x \cdot \dfrac{\text{Cos}\,x}{\text{Sin}\,x}$

$= \boxed{\text{Cos}\,x}$ [$\text{Cotg}\,x = \dfrac{\text{Cos}\,x}{\text{Sin}\,x}$ aldık. (Sonuç 2 den)]

C) $\tan\theta \,.\, \text{Cotg}^2\,\theta = \tan\theta\,(\text{Cotg}\,\theta)^2 = \underbrace{\tan\theta \,.\, \text{Cotg}\,\theta}_{1} \,.\, \text{Cotg}\,\theta$

$= \boxed{\text{Cotg}\,\theta}$ [$\tan\theta \,.\, \text{Cotg}\,\theta = 1$ aldık. (Sonuç 3 den)]

D) $\sqrt{1 - \text{Cos}\,\theta} \cdot \sqrt{1 + \text{Cos}\,\theta} = \sqrt{(1 - \text{Cos}\,\theta)(1 + \text{Cos}\,\theta)}$

$= \sqrt{1 - \text{Cos}^2\,\theta} = \sqrt{\text{Sin}^2\,\theta} = \sqrt{(\text{Sin}\,\theta)^2} = \boxed{|\text{Sin}\,\theta|}$

$(1 - \text{Cos}^2\,\theta = \text{Sin}^2\,\theta$ (uyarıyı bir daha okuyunuz))

E) $\dfrac{\text{Sin}^2 x - 1}{\text{Cos}^2 x} = \dfrac{-(-\text{Sin}^2 x + 1)}{\text{Cos}^2 x} = \dfrac{-(1 - \text{Sin}^2 x)}{\text{Cos}^2 x}$

$= \dfrac{-(\text{Cos}^2 x)}{\text{Cos}^2 x} = \boxed{-1}$

$(1 - \text{Sin}^2 x = \text{Cos}^2 x$ (uyarıyı bir daha okuyunuz)

F) $\text{Sin}^4 x + \text{Sin}^2 x \,.\, \text{Cos}^2 x = \text{Sin}^2 x \underbrace{[\,\text{Sin}^2 x + \text{Cos}^2 x\,]}_{1}$

$= \boxed{\text{Sin}^2 x}$ ($\text{Sin}^2 x + \text{Cos}^2 x = 1$ (Sonuç 6 dan))

NOT : $a^4 + a^2 b^2 = a^2(a^2 + b^2)$ olduğunu çarpanlara ayırma konusundan hatırlayınız. Çarpanlara ayırma konusunu bir daha tekrarlamadan sadeleştirme sorularını yapmayınız.

G) $\frac{\text{Sin }\theta}{\text{Cos }\theta} + \frac{\text{Cos }\theta}{\text{Sin }\theta} = \frac{\text{Sin }\theta}{\text{Cos }\theta} + \frac{\text{Cos }\theta}{\text{Sin }\theta}$

(Sin θ) (Cos θ)

$= \frac{\text{Sin}^2\,\theta + \text{Cos}^2\,\theta}{\text{Sin }\theta\,.\,\text{Cos }\theta} = \frac{1}{\text{Sin }\theta\,.\,\text{Cos }\theta}$

$= \frac{1}{\text{Sin }\theta} \cdot \frac{1}{\text{Cos }\theta} = \text{Cosec }\theta\,.\,\text{Sec }\theta$

[Önce payda eşitlediğimize dikkat ediniz.]

H) $\tan x\,.\,\text{Cotg } x = \frac{\cancel{\text{Sin } x}}{\text{Cos } x} \cdot \frac{\text{Cos } x}{\cancel{\text{Sin } x}} = 1$

[Sonuç 1 ve 2 den veya sadece sonuç 3'ü kullanarak çözüme ulaşırız.]

2. Aşağıdaki trigonometrik ifadeleri sadeleştiriniz.

A) $\text{Sin }\theta\,.\,\text{Sec }\theta\,.\,\text{Cotg }\theta$

B) $\text{Sec}^2\,\theta - \text{Sec}^2\,\theta\,.\,\text{Sin}^2\,\theta$

C) $(1 - \text{Sec}^2\,\theta)(1 - \text{Cosec}^2\,\theta)$

D) $\frac{\text{Cos}^2\,\theta}{1 + \text{Sin }\theta}$

E) $\frac{1 - \text{tg}^2\,x}{1 + \text{tg}^2\,x}$

F) $(1 + \text{Sin }\theta)(\text{Sec }\theta - \text{tg }\theta)$

G) $1 + \text{tg}^2\,\theta - \text{Sin}^2\,\theta - \text{Sin}^2\,\theta\,.\,\text{tg}^2\,\theta$

H) $\text{Cos}^3 x\,.\,\text{Sin } x + \text{Sin}^3\,x\,.\,\text{Cos } x$

İ) $(\text{Sec }\theta + \text{tg }\theta)^2$

J) $\frac{\text{tg }\theta - 1}{1 - \text{Cotg }\theta}$

K) $\frac{\text{Sin }\theta + \text{Cos }\theta}{\text{Sec }\theta + \text{Cosec }\theta}$

L) $\text{tg}^2\,\theta - \text{Sin}^2\,\theta$

M) $\frac{1}{\text{Cos }\theta + \text{Sin }\theta\,.\,\text{tg }\theta}$

N) $\frac{1}{\text{Sin }\theta + \text{Cos }\theta\,.\,\text{Cotg }\theta}$

O) $\frac{1 + \text{Sin }\theta}{\text{Cos }\theta} + \frac{\text{Cos }\theta}{1 + \text{Sin }\theta}$

ÇÖZÜM :

A) $\text{Sin }\theta\,.\,\text{Sec }\theta\,.\,\text{Cotg }\theta = \text{Sin }\theta \cdot \frac{1}{\cancel{\text{Cos }\theta}} \cdot \frac{\cancel{\text{Cos }\theta}}{\text{Sin }\theta} = 1$

B) $\text{Sec}^2\,\theta - \text{Sec}^2\,\theta\,.\,\text{Sin}^2\,\theta = \text{Sec}^2\,\theta(1 - \text{Sin}^2\,\theta)$

$= \text{Sec}^2\,\theta\,.\,\text{Cos}^2\,\theta = \frac{1}{\cancel{\text{Cos}^2\,\theta}} \cdot \cancel{\text{Cos}^2\,\theta} = 1$

C) $(1 - \text{Sec}^2\,\theta)(1 - \text{Cosec}^2\,\theta) = (1 - \frac{1}{\text{Cos}^2\,\theta})(1 - \frac{1}{\text{Sin}^2\,\theta})$

$= (\frac{\text{Cos}^2\,\theta - 1}{\text{Cos}^2\,\theta})(\frac{\text{Sin}^2\,\theta - 1}{\text{Sin}^2\,\theta})$

$= \frac{-(1 - \text{Cos}^2\,\theta)}{\text{Cos}^2\,\theta} \cdot \frac{-(1 - \text{Sin}^2\,\theta)}{\text{Sin}^2\,\theta}$

$= \frac{\cancel{-(\text{Sin}^2\,\theta)}}{\cancel{\text{Cos}^2\,\theta}} \cdot \frac{\cancel{-(\text{Cos}^2\,\theta)}}{\cancel{\text{Sin}^2\,\theta}} = (-1)(-1) = +1$

D) $\frac{\text{Cos}^2\,\theta}{1 + \text{Sin}\theta} = \frac{1 - \text{Sin}^2\,\theta}{1 + \text{Sin }\theta} = \frac{(1 - \text{Sin}\theta)\cancel{(1 + \text{Sin }\theta)}}{\cancel{1 + \text{Sin }\theta}}$

$= 1 - \text{Sin }\theta$

E) $$\frac{1-tg^2 x}{1+tg^2 x} = \frac{1-(\frac{\sin x}{\cos x})^2}{1+(\frac{\sin x}{\cos x})^2} = \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$$

$$= \frac{\cos^2 x - \sin^2 x}{\underbrace{\cos^2 x + \sin^2 x}_{1}} = \cos^2 x - \sin^2 x$$

F) $(1+\sin\theta)(\sec\theta - tg\theta)$

$$= (1+\sin\theta)\cdot(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta})$$

$$= (1+\sin\theta)\cdot(\frac{1-\sin\theta}{\cos\theta})$$

$$= \frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta} = \frac{1-\sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta} = \cos\theta$$

G) $1 + tg^2\theta - \sin^2\theta - \sin^2\theta . tg^2\theta$

$$= (1+tg^2\theta) - \sin^2\theta(1+tg^2\theta)$$

$$= (1+tg^2\theta)(1-\sin^2\theta)$$

$$= (1+\frac{\sin^2\theta}{\cos^2\theta})(1-\sin^2\theta)$$

$$= (\frac{\overbrace{\cos^2\theta + \sin^2\theta}^{1}}{\cos^2\theta})(\cos^2\theta) = \frac{1}{\cos^2\theta}\cdot\cos^2\theta$$

$$= 1$$

[$1 + tg^2\theta = \sec^2\theta$ olduğunu görebildiniz mi ?]

H) $\cos^3 x . \sin x + \sin^3 x . \cos x$

$$= \sin x . \cos x \underbrace{[\cos^2 x + \sin^2 x]}_{1} = \sin x . \cos x$$

İ) $$(\sec\theta + tg\theta)^2 = (\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta})^2 = (\frac{1+\sin\theta}{\cos\theta})^2$$

$$= \frac{(1+\sin\theta)^2}{\cos^2\theta} = \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1+\sin\theta)\cancel{(1+\sin\theta)}}{(1-\sin\theta)\cancel{(1+\sin\theta)}}$$

$$= \frac{1+\sin\theta}{1-\sin\theta}$$

J) $$\frac{tg\theta - 1}{1-\text{Cotg}\,\theta} = \frac{\frac{\sin\theta}{\cos\theta} - 1}{1 - \frac{\cos\theta}{\sin\theta}} = \frac{\frac{\sin\theta - \cos\theta}{\cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta}}$$

$$= \frac{\cancel{\sin\theta - \cos\theta}}{\cos\theta}\cdot\frac{\sin\theta}{\cancel{\sin\theta - \cos\theta}} = \frac{\sin\theta}{\cos\theta} = tg\,\theta$$

K) $$\frac{\sin\theta + \cos\theta}{\sec\theta + \text{Cosec}\,\theta} = \frac{\sin\theta + \cos\theta}{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}$$
$(\sin\theta)$ $(\cos\theta)$

$$= \frac{\sin\theta + \cos\theta}{\frac{\sin\theta + \cos\theta}{\sin\theta . \cos\theta}} = \frac{\sin\theta + \cos\theta}{1}\cdot\frac{\sin\theta . \cos\theta}{\sin\theta + \cos\theta}$$

$= \sin\theta . \cos\theta$

L) $\tg^2\theta - \Sin^2\theta = \dfrac{\Sin^2\theta}{\Cos^2\theta} - \Sin^2\theta$

$= \dfrac{\Sin^2\theta - \Sin^2\theta \cdot \Cos^2\theta}{\Cos^2\theta} = \dfrac{\Sin^2\theta(1 - \Cos^2\theta)}{\Cos^2\theta}$

$= \dfrac{\Sin^2\theta \cdot \Sin^2\theta}{\Cos^2\theta} = \dfrac{\Sin^2\theta}{\Cos^2\theta} \cdot \Sin^2\theta = \tg^2\theta \cdot \Sin^2\theta$

M) $\dfrac{1}{\Cos\theta + \Sin\theta \cdot \tg\theta} = \dfrac{1}{\Cos\theta + \Sin\theta \cdot \dfrac{\Sin\theta}{\Cos\theta}}$

$= \dfrac{1}{\Cos\theta + \dfrac{\Sin^2\theta}{\Cos\theta}} = \dfrac{1}{\dfrac{\Cos^2\theta + \Sin^2\theta}{\Cos\theta}}$

$= \dfrac{\Cos\theta}{\underbrace{\Cos^2\theta + \Sin^2\theta}_{1}} = \Cos\theta$

N) $\dfrac{1}{\Sin\theta + \Cos\theta \cdot \Cot\theta} = \dfrac{1}{\Sin\theta + \Cos\theta \cdot \dfrac{\Cos\theta}{\Sin\theta}}$

$= \dfrac{1}{\dfrac{\Sin^2\theta + \Cos^2\theta}{\Sin\theta}} = \Sin\theta$

O) $\dfrac{1 + \Sin\theta}{\Cos\theta} + \dfrac{\Cos\theta}{1 + \Sin\theta} = \dfrac{(1 + \Sin\theta)^2 + \Cos^2\theta}{\Cos\theta(1 + \Sin\theta)}$

$= \dfrac{1 + 2\Sin\theta + \overbrace{\Sin^2\theta + \Cos^2\theta}^{1}}{\Cos\theta(1 + \Sin\theta)} = \dfrac{2 + 2\Sin\theta}{\Cos\theta(1 + \Sin\theta)}$

$= \dfrac{\cancel{2(1 + \Sin\theta)}}{\Cos\theta\cancel{(1 + \Sin\theta)}} = \dfrac{2}{\Cos\theta} = 2 \cdot \dfrac{1}{\Cos\theta} = 2 \cdot \Sec\theta$

3. $x = A \cdot \Cos\theta$ ve $y = A \cdot \Sin\theta$ ise, $x^2 + y^2$ toplamının A cinsinden eşiti nedir ?

ÇÖZÜM :

$x^2 + y^2 = (A \cdot \Cos\theta)^2 + (A \cdot \Sin\theta)^2 = A^2 \cdot \Cos^2\theta + A^2 \cdot \Sin^2\theta$

$= A^2 \underbrace{(\Cos^2\theta + \Sin^2\theta)}_{1} = A^2$

4. $A = x \cdot \Cos\alpha + y \cdot \Sin\alpha$ ve $B = x \cdot \Sin\alpha - y \cdot \Cos\alpha$ ise, $A^2 + B^2$ toplamının x ve y cinsinden eşiti nedir ?

ÇÖZÜM :

$A^2 = (x \Cos\alpha + y \Sin\alpha)^2$

$= (x \Cos\alpha)^2 + 2 \cdot (x \Cos\alpha) \cdot (y \Sin\alpha) + (y \Sin\alpha)^2$

$= x^2 \cdot \Cos^2\alpha + 2xy \cdot \Cos\alpha \cdot \Sin\alpha + y^2 \Sin^2\alpha$

$B^2 = (x \Sin\alpha - y \Cos\alpha)^2$

$= (x \Sin\alpha)^2 - 2 \cdot (x \Sin\alpha)(y \Cos\alpha) + (y \Cos\alpha)^2$

$= x^2 \Sin^2\alpha - 2xy \cdot \Sin\alpha \Cos\alpha + y^2 \Cos^2\alpha$

O halde;

$A^2 + B^2 = (x^2 \Cos^2\alpha + 2xy \cancel{\Cos\alpha \Sin\alpha} + y^2 \Sin^2\alpha)$

$+ (x^2 \Sin^2\alpha - 2xy \cancel{\Sin\alpha \Cos\alpha} + y^2 \Cos^2\alpha)$

$= x^2 \Cos^2\alpha + x^2 \Sin^2\alpha + y^2 \Sin^2\alpha + y^2 \Cos^2\alpha$

$= x^2 (\Cos^2\alpha + \Sin^2\alpha) + y^2 (\Sin^2\alpha + \Cos^2\alpha)$

$= x^2 + y^2$

5. A) $0° < \alpha < 90°$ ve $\Sin\alpha = \dfrac{3}{5}$ ise, $\Cos\alpha$ nedir ?

B) $0° < \alpha < 90°$ ve $\Cos\alpha = \dfrac{1}{3}$ ise, $\Sin\alpha$ nedir ?

C) $0° < \alpha < 90°$ ve $\text{tg}\,\alpha = 4$ ise, $\text{Cotg}\,\alpha$ nedir ?

D) $0° < \alpha < 90°$ ve $\text{Cotg}\,\alpha = \pi$ ise, $\text{tg}\,\alpha$ nedir ?

E) $0° < \alpha < 90°$ ve $\text{Sec}\,\alpha = 3$ ise, $\text{Cos}\,\alpha$ nedir ?

F) $0° < \alpha < 90°$ ve $\text{Sin}\,\alpha = \frac{1}{5}$ ise, $\text{Cosec}\,\alpha$ nedir ?

ÇÖZÜM :

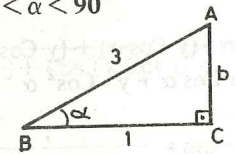
A) $\text{Sin}^2\alpha + \text{Cos}^2\alpha = 1$ idi. O halde $\text{Cos}^2\alpha = 1 - \text{Sin}^2\alpha$

$= 1 - [\frac{3}{5}]^2 = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow \text{Cos}^2\alpha = \frac{16}{25}$

iken, $\text{Cos}\,\alpha = \frac{4}{5}$ veya $\text{Cos}\,\alpha = -\frac{4}{5}$ dir. Ancak

$0° < \alpha < 90°$ olduğundan, $\text{Cos}\,\alpha = \frac{4}{5}$ seçilir.

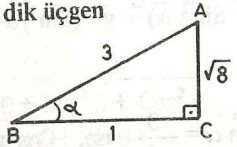
B) $0° < \alpha < 90°$



$\text{Cos}\,\alpha = \frac{1}{3}$ idi. Bunu taslak bir dik üçgende gösterirsek, Pisagor teoreminden diğer dik kenarıda hesaplamamız gerekir.

$b^2 + 1^2 = 3^2$, $b^2 = 8$, $b = \sqrt{8}$

O halde dik üçgen



haline gelir. Buradan

$\text{Sin}\alpha = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$ dir.

C) $\boxed{\text{Tg}\,\alpha \,.\, \text{Cotg}\,\alpha = 1}$ idi. $4 \,.\, \text{Cotg}\alpha = 1 \Rightarrow \text{Cotg}\,\alpha = \frac{1}{4}$ dür.

D) $\boxed{\text{Cotg}\,\alpha \,.\, \text{tg}\,\alpha = 1}$ idi. $\pi \,.\, \text{tg}\,\alpha = 1 \Rightarrow \text{tg}\,\alpha = \frac{1}{\pi}$ dir.

NOT : Burada kullanılan π sayıdır, açı ölçüsü olarak kullanılan radyan değildir. $\pi \cong 3{,}14159265358979323846$ .....

E) $\text{Sec}\,\alpha = \frac{1}{\text{Cos}\,\alpha}$ idi. $\text{Cos}\,\alpha = \frac{1}{\text{Sec}\alpha}$ ve $\text{Cos}\alpha = \frac{1}{3}$ dür.

F) $\text{Cosec}\,\alpha = \frac{1}{\text{Sin}\alpha}$ idi. $\text{Cosec}\,\alpha = \frac{1}{\frac{1}{5}} = 5$ dir.

6. Aşağıdakileri sadeleştiriniz.

A) $\frac{\text{tg}\,20}{\text{Cotg}\,70} + \frac{\text{Sin}\,80}{\text{Cos}\,10}$

B) $\frac{\text{Cos}\,100}{\text{Sin}\,(-10)} + \frac{\text{tg}\,75}{\text{Cotg}\,15} - \frac{\text{Sin}\,(70 + 5\alpha)}{\text{Cos}\,(20 - 5\alpha)}$

C) $\frac{\text{Sin}\,(83 + \frac{93\alpha}{7})}{\text{Cos}\,(-\frac{93\alpha}{7} + 7)} + \frac{\text{Sin}^2\,51}{\text{Cos}^2\,39} + (\frac{\text{tg}\,35}{\text{Cotg}\,55})^2 - 3\text{tg}10.\text{Cotg}10$

D) $\text{Sin}^2 40 + \text{Sin}^2 50 + \text{Cos}^2 70 + \text{Cos}^2 20$

ÇÖZÜM :

A) $\text{tg}20° = \text{Cotg}70°$ ve $\text{Sin}80° = \text{Cos}10°$ olduğunu biliyoruz. (Sonuç 7)

$\frac{\text{tg}\,20°}{\text{Cotg}\,70°} + \frac{\text{Sin}\,80°}{\text{Cos}\,10°} = 1 + 1 = 2$

B) $\frac{\text{Cos}100°}{\text{Sin}\,(-10°)} + \frac{\text{tg}\,75°}{\text{Cotg}\,15°} - \frac{\text{Sin}\,(70 + 5\alpha)}{\text{Cos}\,(20 - 5\alpha)}$

$= 1 + 1 - 1 = 1$ (Sonuç 7)

C) $\dfrac{\operatorname{Sin}(83+\frac{93\alpha}{7})}{\operatorname{Cos}(\frac{93\alpha}{7}+7)} + (\dfrac{\operatorname{Sin}51}{\operatorname{Cos}39})^2 + (\dfrac{\operatorname{tg}35^\circ}{\operatorname{Cotg}55^\circ})^2$

$-3\underbrace{\operatorname{tg}10 \cdot \operatorname{Cotg}10}_{1} = 1+1+1-3 = 0$

[Sonuç 7 ve sonuç 3]

D) $(\operatorname{Sin}^2 40 + \operatorname{Sin}^2 50) + (\operatorname{Cos}^2 70 + \operatorname{Cos}^2 20)$

$= (\operatorname{Sin}^2 40 + \operatorname{Cos}^2 40) + (\operatorname{Cos}^2 70 + \operatorname{Sin}^2 70)$

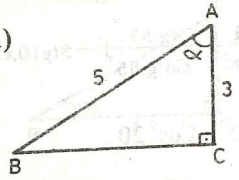
$= 1 + 1 = 2$

[$\operatorname{Sin}50 = \operatorname{Cos}40 \Rightarrow \operatorname{Sin}^2 50 = \operatorname{Cos}^2 40$ ve

$\operatorname{Cos}20 = \operatorname{Sin}70 \Rightarrow \operatorname{Cos}^2 20 = \operatorname{Sin}^2 70$ dir.] (Sonuç 7)

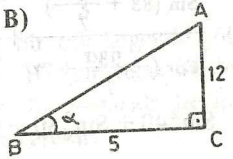
7. Aşağıdaki verilen dik üçgenlerde istenilen trigonometrik or
ları bulunuz.

A)



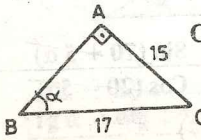
Sin α = ?
tg α = ?
Cotg α = ?

B)



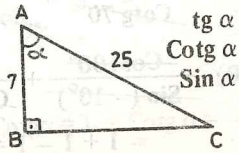
Cos α = ?
Sin α = ?
Cotg α = ?

C)



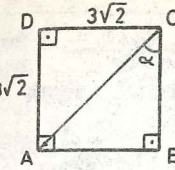
Cos α = ?
tg α = ?

D)



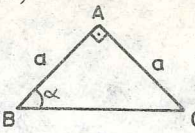
tg α =
Cotg α =
Sin α =

E)



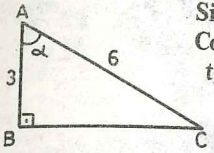
tg α = ?

F)



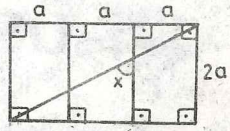
tg α = ?
Sin α = ?

G)



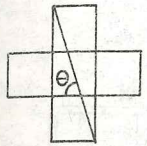
Sin α = ?
Cos α = ?
tg α = ?

H)



tg x = ?
Cotgx = ?

İ)



Şekilde kareler eş ise;
tg θ = ?

YANITLAR :

A) Sin α = 4/5
tg α = 4/3
Cotg α = 3/4

B) Cos α = 5/13
Sin α = 12/13
Cotg α = 5/12

C) Cos α = 8/17
tg α = 15/8

D) tg α = 24/7
Cotg α = 7/24
Sin α = 24/25

E) tg α = 1

F) tg α = 1
Sin α = $\frac{\sqrt{2}}{2}$

G) Sin α = $\frac{\sqrt{3}}{2}$
Cos α = 1/2
tg α = $\sqrt{3}$

H) tg x = 3/2
Cotgx = 2/3

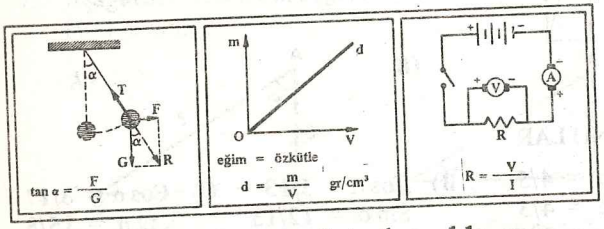
İ) tg θ = 3

büyük
yayınları

YENİ SINAV SİSTEMİNE GÖRE HAZIRLANMIŞTIR

ÖSS
İÇİN

FİZİK



• ÖSS konularının detaylı açıklaması
• 350 Çözümlü örnek test
• 460 Cevaplı test

Nurettin Ceran Necati Tanrıöver

4. BÖLÜM

PERİYODİK FONKSİYONLAR
TRİGONOMETRİK ve TERS TRİGONOMETRİK FONKSİYONLARIN GRAFİKLERİ

I. Periyod : $f:A \to B$ fonksiyonunda $\forall x \in A$ için $f(x+T) = f(x)$ koşulunu sağlayan en küçük pozitif T değerine f fonksiyonunun periyodu denir.

Periyod Bulunması :

A) $f(x) = a \cdot \text{Sin}^m(bx+c)$
$f(x) = a \cdot \text{Cos}^m(bx+c)$
$f(x) = a \cdot \text{Sec}^m(bx+c)$
$f(x) = a \cdot \text{Cosec}^m(bx+c)$
} fonksiyonları için,

1) m tek ise, periyod $T = \frac{2\pi}{b}$

2) m çift ise, periyod $T = \frac{\pi}{b}$ dir.

B) $f(x) = a \cdot \tan^m(bx+c)$
$f(x) = a \cdot \text{Cotg}^m(bx+c)$
} fonksiyonları için

Periyod $T = \frac{\pi}{b}$ dir.

C) Toplam şeklindeki trigonometrik fonksiyonların periyodları; periyodlarının OKEK'i alınarak bulunur.

SORU: Aşağıdaki fonksiyonların periyodlarını bulunuz.

A) $y = \text{Sin}(2x + 1)$ B) $y = \text{Cos}(5x - 4)$

C) $y = 7 \cdot \text{Sin}^2(3x - 1)$ D) $y = \text{tg}(3x - 1)$

E) $y = \text{Cos}^3\left(\frac{\pi}{2}x + 4\right)$ F) $y = -4 \cdot \text{Sin}^9\left(\frac{x}{2} + 5\right)$

ÇÖZÜM :

A) $T = \frac{2\pi}{2} = \pi$ B) $T = \frac{2\pi}{5}$ C) $T = \frac{\pi}{3}$

D) $T = \frac{\pi}{3}$ E) $T = \frac{2\pi}{\frac{\pi}{2}} = 4$ F) $T = \frac{2\pi}{\frac{1}{2}} =$

SORU : Aşağıdaki fonksiyonların periyodlarını bulunuz.

A) $y = \text{Sin}(5x - 1) + \text{Cos}(2x + 3)$

B) $y = \text{Sin}^2(4x - 1) + \text{Cos}^3(6x - 2)$

C) $y = \text{Sin}^3(5x + 1) + \text{tg}^2(x - 4)$

D) $y = \text{Cos}^6(2x - 1) + \text{Sin}(2x + 3)$

ÇÖZÜM :

A) $\text{Sin}(5x - 1)$; $T_1 = \frac{2\pi}{5}$

$\text{Cos}(2x + 3)$; $T_2 = \frac{2\pi}{2}$

} Paydaları eşitlenir. $\Rightarrow$

$T_1 = \frac{4\pi}{10}$

$T_2 = \frac{10\pi}{10}$

} Payların Okek'i alınır. $\Rightarrow$ } $T = \frac{20\pi}{10} = 2\pi$ bulunu

B) $T_1 = \frac{\pi}{4}$

$T_2 = \frac{2\pi}{6} = \frac{\pi}{3}$

} $\Rightarrow$ $T_1 = \frac{3\pi}{12}$

$T_2 = \frac{4\pi}{12}$

} $\Rightarrow$

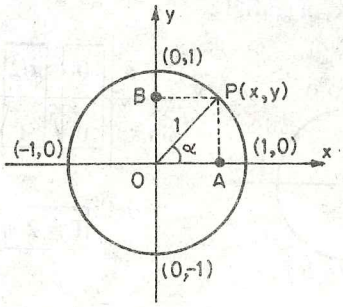
$\Rightarrow T = \frac{12\pi}{12} = \pi$ dir.

C) $T_1 = \frac{2\pi}{5}$

$T_2 = \frac{\pi}{1}$

} $T = 2\pi$ dir. D) $T = \pi$ dir.

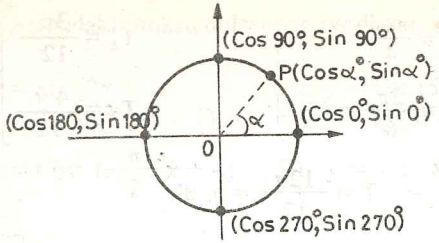
II. **Sinüs ve Kosinüs Fonksiyonları :**



$\text{Sin}\,\alpha = \frac{|AP|}{|OP|} = \frac{|AP|}{1} = |AP| = y$

$\text{Cos}\,\alpha = \frac{|OA|}{|OP|} = \frac{|OA|}{1} = |OA| = x$

* $\alpha \in R$ ye birim çember üzerinde karşılık gelen nokta P olsun. P noktasının apsisi $\text{Cos}\,\alpha$, ordinatı $\text{Sin}\,\alpha$ dir. P noktası P (x, y) yerine P ($\text{Cos}\,\alpha$, $\text{Sin}\,\alpha$) olarak yazılabilir.

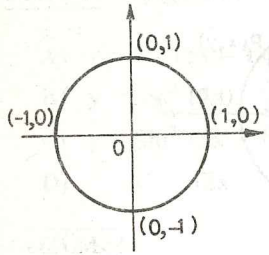


Şekilden ;

$\text{Cos}0 = 1$	$\text{Cos}90 = 0$	$\text{Cos}180 = -1$	$\text{Cos}270 = 0$
$\text{Sin}0 = 0$	$\text{Sin}90 = 1$	$\text{Sin}180 = 0$	$\text{Sin}270 = -1$

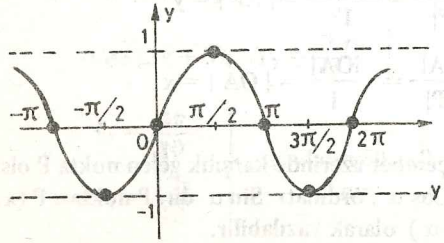
bulunabilir.

* f (x) = Sin x in grafiği :



x	0	$\pi/2$	π	$3\pi/2$	2π
Sin x	0	1	0	-1	0

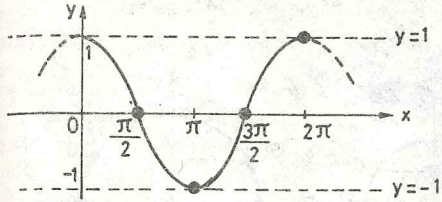
$T = 2\pi$



$-1 \leq \text{Sin}\,\alpha \leq 1$

* f (x) = Cos x in grafiği :

x	0	$\pi/2$	π	$3\pi/2$	2π
Cos x	1	0	-1	0	1



$T = 2\pi$

$-1 \leq \text{Cos}\alpha \leq 1$

ÖRNEK: Aşağıdakileri birim çemberi inceleyerek yapmız.

1) α açısı I. bölgede $0°$ den $90°$ ye artarken $\text{Sin}\,\alpha$ nasıl değişir ? [0 dan 1'e artar.]

2) α açısı I. bölgede $0°$ den $90°$ ye artarken $\text{Cos}\,\alpha$ nasıl değişir ? [1 den 0'a azalır.]

3) α açısı II. bölgede $90°$ den $180°$ ye artarken $\text{Sin}\,\alpha$ nasıl değişir ? [1 den 0'a azalır.]

4) α açısı II. bölgede $90°$ den $180°$ ye artarken $\text{Cos}\,\alpha$ nasıl değişir ? [0 dan -1'e azalır.]

5) α açısı III. bölgede $180°$ den $270°$ ye artarken $\text{Sin}\,\alpha$ nasıl değişir ? [0 dan -1'e azalır.]

6) α açısı III. bölgede $180°$ den $270°$ ye artarken $\text{Cos}\,\alpha$ nasıl değişir ? [-1 den 0'a artar.]

7) α açısı IV. bölgede $270°$ den $360°$ ye artarken $\text{Sin}\,\alpha$ nasıl değişir ? [-1 den 0'a artar.]

8) α açısı IV. bölgede $270°$ den $360°$ ye artarken $\text{Cos}\,\alpha$ nasıl değişir ? [0 dan 1'e artar.]

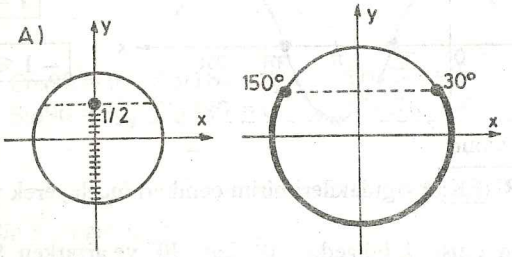
SORU: Aşağıdaki eşitsizlikleri birim çemberde gösteriniz.

A) $\sin\alpha \leq \frac{1}{2}$ B) $\cos\alpha \leq \frac{\sqrt{2}}{2}$ C) $\sin\alpha \geq \frac{\sqrt{3}}{2}$

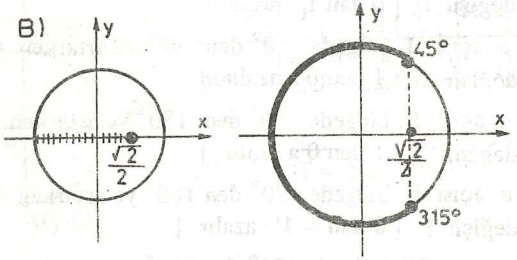
D) $\cos\alpha \geq \frac{1}{2}$ E) $\sin\alpha \leq -\frac{1}{2}$ F) $\cos\alpha \geq -\frac{1}{2}$

ÇÖZÜM:

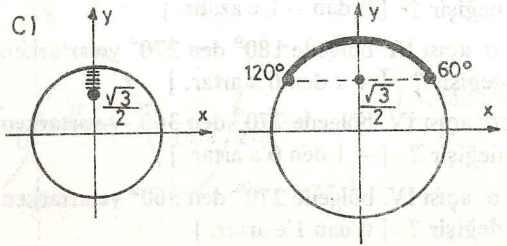
A)



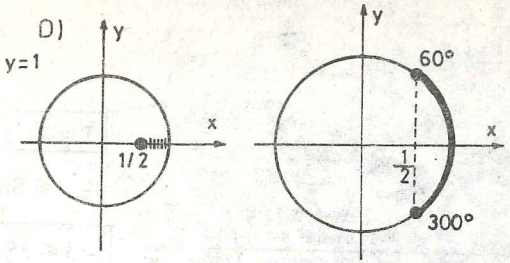
B)



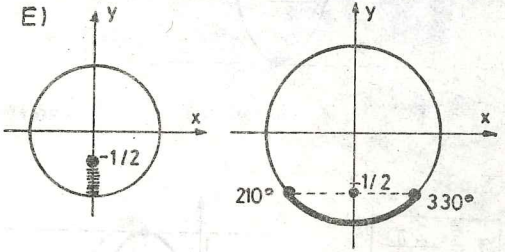
C)



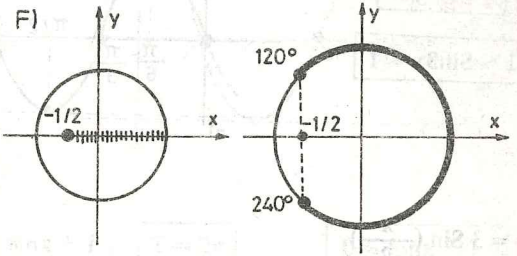
D)



E)



F)



SORU: Aşağıdaki grafikleri çiziniz.

A) $y = 3\,\mathrm{Sin}x$ B) $y = \mathrm{Sin}\,3x$ C) $y = 3\,\mathrm{Sin}\,(\frac{x}{2})$

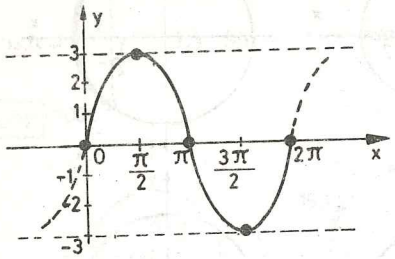
D) $y = 4\,\mathrm{Cos}x$ E) $y = 4\,\mathrm{Cos}\,(\frac{x}{2})$ F) $y = \mathrm{Sin}x + 1$

G) $y = \mathrm{Cos}x - 2$ H) $y = 2\,\mathrm{Cos}x + 1$ İ) $y = 3\,\mathrm{Cos}x - 2$

J) $y = \mathrm{Sin}x + \mathrm{Cos}x$

ÇÖZÜM :

A)



$T = 2\pi$

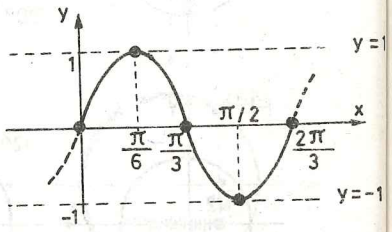
$-1 \leq \text{Sinx} \leq 1$

$-3 \leq 3\text{Sinx} \leq 3$

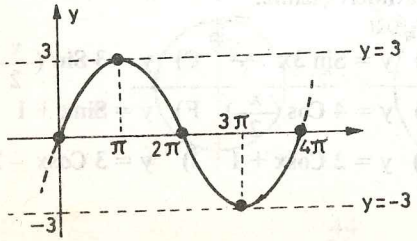
B) $y = \text{Sin}3x$

$T = \frac{2\pi}{3}$

$-1 \leq \text{Sin}3x \leq 1$



C) $y = 3\ \text{Sin}\left(\frac{x}{2}\right)$



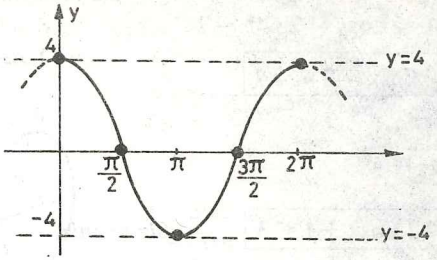
$T = 4\pi$

$-3 \leq 3\text{Sin}\left(\frac{x}{2}\right) \leq 3$

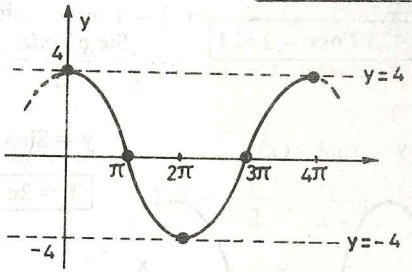
D) $y = 4\text{Cosx}$

$-4 \leq 4\ .\ \text{Cosx} \leq 4$

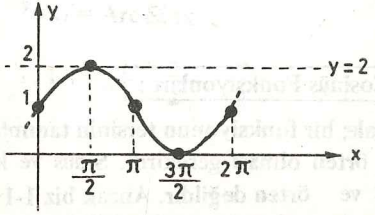
$T = 2\pi$



E) $y = 4\text{Cos}\left(\frac{x}{2}\right)$, $T = 4\pi$, $-4 \leq 4\text{Cos}\left(\frac{x}{2}\right) \leq 4$



F) $y = \text{Sinx} + 1$, $T = 2\pi$, $0 \leq \text{Sinx} + 1 \leq 2$



$$\begin{cases} -1 \leq \text{Sinx} \leq 1 \\ -1+1 \leq \text{Sinx}+1 \leq 1+1 \\ 0 \leq \text{Sinx}+1 \leq 2 \end{cases}$$

* (y = Sinx fonksiyonunu 1 birim yukarı çektik.)

G) $\boxed{T = 2\pi}$

$-1 \leq \text{Cosx} \leq 1$ *Siz deneyiniz.*

$-1-2 \leq \text{Cosx} - 2 \leq 1-2$

$\boxed{-3 \leq \text{Cosx} - 2 \leq -1}$

H) $\boxed{T = 2\pi}$

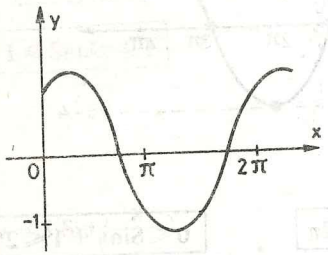
$\boxed{-1 \leq 2\,\text{Cosx} + 1 \leq 3}$ *Siz çiziniz.*

İ) $\boxed{T = 2\pi}$

$\boxed{-5 \leq 3\,\text{Cosx} - 2 \leq 1}$ *Siz çiziniz.*

J) $y = \text{Sinx} + \text{Cosx}$

$\boxed{T = 2\pi}$

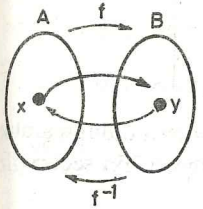


III. Arc Sinüs ve Arc Cosinüs Fonksiyonları :

* Lise 1 den hatırlarsak; bir fonksiyonun tersinin tanımlanabilmesi için 1– 1 ve örten olması gerekirdi. Sinüs ve kosinüs fonksiyonları 1 – 1 ve örten değildir. Ancak biz 1-1 ve örten oldukları aralıklarda terslerini tanımlayabiliriz.

* $f(x) = \text{Sin}\,x$ fonksiyonunun ters fonksiyonu $\text{Sin}^{-1}(x)$ [$= \text{Cosx}$] [$\text{Cos}^{-1}(x)$]

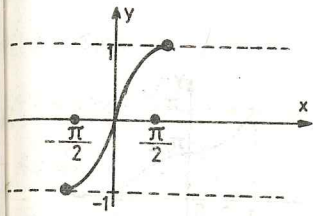
veya Arc Sinx olarak gösterilir.
[Arc Cosx]



$f(x) = y$
$f^{-1}(y) = x$ } olduğunu hatırlarsak,

$y = \text{Sinx}$ ise, $x = \text{Arc Siny}$
$y = \text{Cosx}$ ise, $x = \text{Arc Cosy}$ diyebiliriz.

Şimdi $y = \text{Sinx}$ için $1-1$ ve örten bir aralık bulalım. Genel uygulama $[-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow [-1, 1]$ dir.

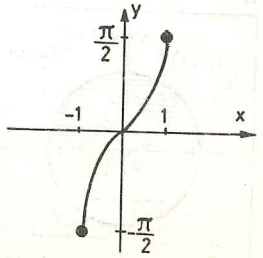


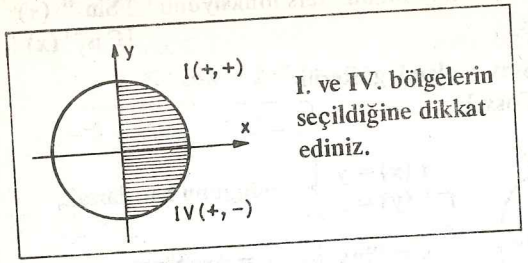
$f(x) = \text{Sinx}$,

$f : [-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow [-1, 1]$

$f(x) = \text{Arc Sinx}$,

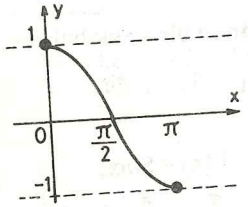
$f : [-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$





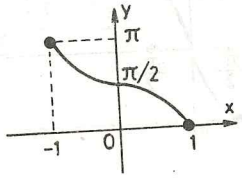
I. ve IV. bölgelerin seçildiğine dikkat ediniz.

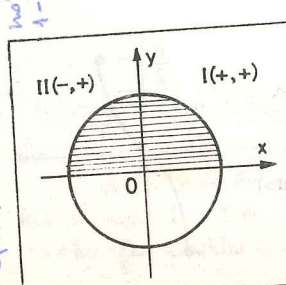
* f (x) = Cos x fonksiyonunun 1 – 1 ve örten olduğu aralıklardan birisini seçelim. (Elbette genel uygulamayı seçeceğiz)



f (x) = Cosx
f : [0, π] → [– 1, 1]

f (x) = Arc Cosx
f : [– 1, 1] → [0, π]





I. ve II. bölgelerin seçildiğine dikkat ediniz.

UYARI : $f : A \to B$, $1-1$ ve örten ise, $f(a) = b$ iken, $f^{-1}(b) = a$ olduğunu hatırlayınız.

A) $\text{Sin}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ ise, $\text{Arc Sin}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

[$\text{Arc Sin}\left(\frac{\sqrt{3}}{2}\right)$ yi hangi açının Sinüsü $\frac{\sqrt{3}}{2}$ dir ? Şeklinde okuyunuz.]

B) $\text{Cos}\left(\frac{\pi}{3}\right) = \frac{1}{2}$ ise, $\text{Arc Cos}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ olur.

SORU : Aşağıdakilerin eşitlerini bulunuz. (Daha önce tanımladığımız bölgelere dikkat ediniz.)

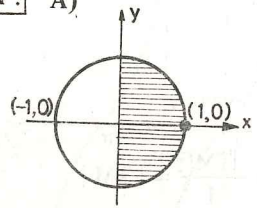
A) Arc Sin (0) B) $\text{Arc Sin}\left(\frac{1}{2}\right)$ C) $\text{Arc Sin}\left(\frac{\sqrt{2}}{2}\right)$

D) Arc Sin (– 1) E) $\text{Arc Sin}\left(-\frac{1}{2}\right)$ F) Arc Cos (0)

G) $\text{Arc Cos}\left(\frac{\sqrt{2}}{2}\right)$ H) $\text{Arc Cos}\left(\frac{\sqrt{3}}{2}\right)$ İ) $\text{Arc Cos}\left(-\frac{1}{2}\right)$

J) $\text{Arc Cos}\left(-\frac{\sqrt{3}}{2}\right)$

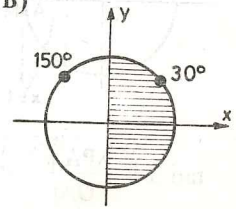
ÇÖZÜM : A)



Yanıt 0

(Neden π değil ?)

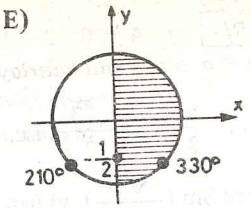
B)



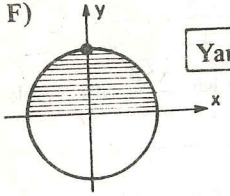
Yanıt π/6

(Neden 5π/6 değil ?)

C) $\pi/4$ D) $3\pi/2$ $(-\pi/2)$ E)



F)

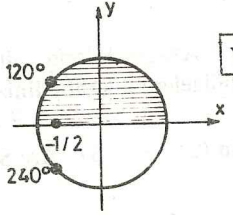


Yanıt $\pi/2$

Yanıt $11\pi/6$ $(-\pi/6)$

(Neden $7\pi/6$ değil ?)

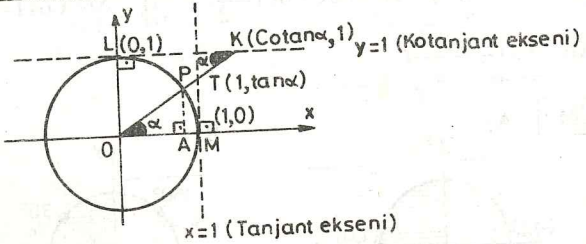
G) $\pi/4$ H) $\pi/6$ İ)



Yanıt 2π

J) $5\pi/6$

IV. Tanjant - Kotanjant Fonksiyonları :



$$\tan\alpha = \frac{|PA|}{|OA|} = \frac{|TM|}{|OM|} = \frac{|TM|}{1} = |TM|$$

$$\text{Cotan}\,\alpha = \frac{|OA|}{|AP|} = \frac{|LK|}{|OL|} = \frac{|LK|}{1} = |LK|$$

SORU: Aşağıdakileri birim çemberi kullanarak yanıtlayınız.

A) α açısı $0°$ den $90°$ ye artarken $\tan\alpha$ nasıl değişir ?
[O dan $+\infty$ 'a artar.]

B) α açısı $0°$ den $90°$ ye artarken Cotg α nasıl değişir ?
[$+\infty$ dan 0'a azalır.]

C) α açısı $90°$ den $180°$ ye artarken $\tan\alpha$ nasıl değişir ?
[$-\infty$ dan 0'a artar.]

D) α açısı $90°$ den $180°$ ye artarken Cotg α nasıl değişir ?
[O dan $-\infty$ 'a azalır.]

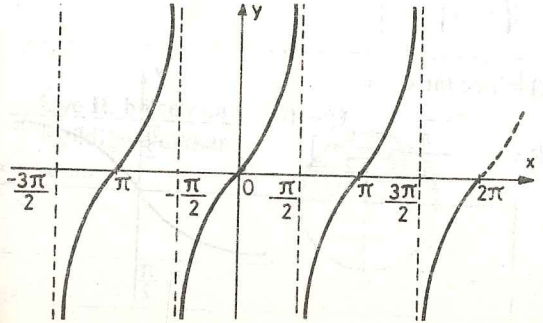
E) α açısı $180°$ den $270°$ ye artarken $\tan\alpha$ nasıl değişir ?
[0 dan $+\infty$ 'a artar.]

F) α açısı $180°$ den $270°$ ye artarken Cotg α nasıl değişir ?
[$+\infty$ dan 0'a azalır.]

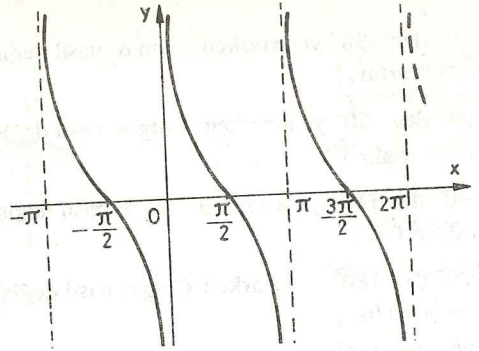
G) α açısı $270°$ den $360°$ ye artarken $\tan\alpha$ nasıl değişir ?
[$-\infty$ dan 0'a artar.]

H) α açısı $270°$ den $360°$ ye artarken Cotg α nasıl değişir ?
[0 dan $-\infty$ 'a azalır.]

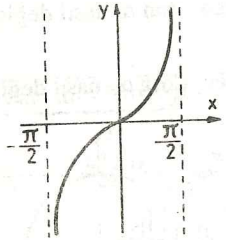
$f(x) = \tan x$ in grafiği ;



* f (x) = Cotg x in grafiği ;



* tanjant ve kotanjant fonksiyonlarının 1 – 1 ve örten ol
dıkları görülüyor. Yine 1 – 1 ve örten oldukları birer ar
seçerek ters fonksiyonlarını tanımlayacağız.

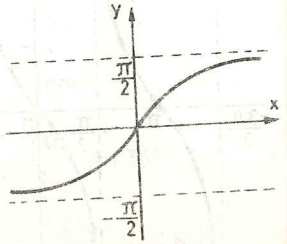


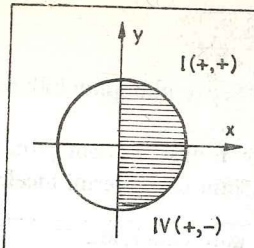
f (x) = tan x

$f : [-\frac{\pi}{2} , \frac{\pi}{2}] \to R$

* f (x) = Arc tanx

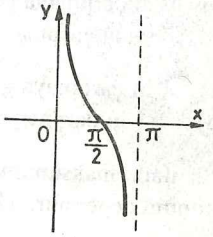
$f : R \to [-\frac{\pi}{2} , \frac{\pi}{2}]$

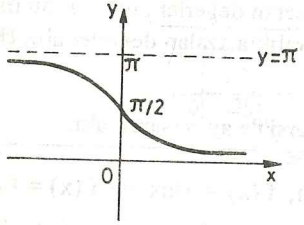




I. ve IV. bölgelerin seçildiğine dikkat ediniz.

f (x) = Cotg x f : [0, π] → R

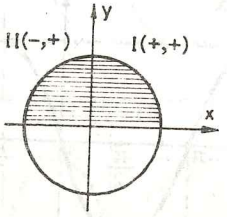




f (x) = Arc Cotg x

f : R → [0, π]

I. ve II. bölgelerin seçildiğine dikkat ediniz.



V. Secant ve Kosecant Fonksiyonları :

$\frac{1}{\text{Sin }\alpha} = \text{Cosec }\alpha$ ve $\frac{1}{\text{Cos }\alpha} = \text{Sec }\alpha$ olduğunu biliyoruz.

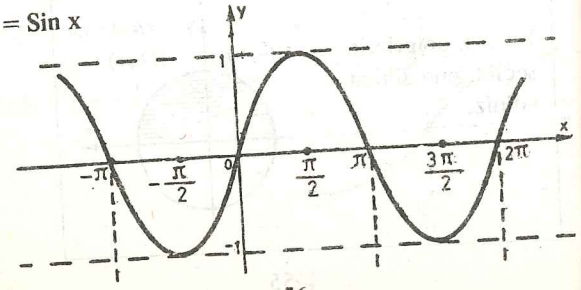
$f(x) = \text{Sec}\alpha$ ve $f(x) = \text{Cosec } x$ fonksiyonlarını çizmek için bir fonksiyonun çarpmaya göre tersinin özelliklerini inceleyelim

NOT : Bir fonksiyonun çarpmaya göre tersi için ;

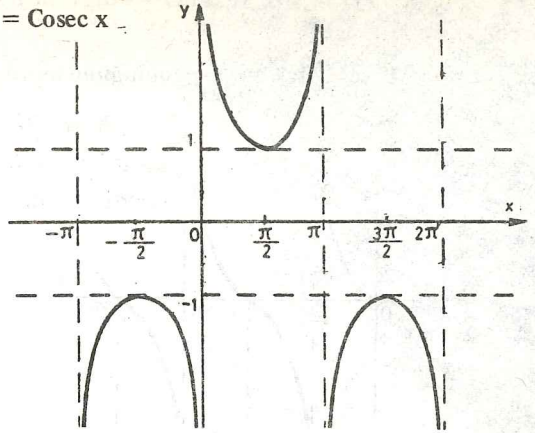
1. a) 0 ın çarpmaya göre tersi $\mp\infty$ dur.
   b) $\mp\infty$ ın çarpmaya göre tersi 0 dır.
2. a) 1 in çarpmaya göre tersi 1 dir.
   b) -1 in çarpmaya göre tersi -1 dir.
3. Bir ifade maksimum değer aldığında çarpmaya göre tersi m nimum değer alır. (Karşıtı da doğru)
4. Bir ifade belli bir aralıkta artan değerler alıyorsa; bu ifaden çarpmaya göre tersi aynı aralıkta azalan değerler alır. (Karş da doğrudur.)
5. Bir ifade, çarpmaya göre tersi ile aynı işareti alır.

A) Şimdi $-\pi \leq x \leq 2\pi$ iken, $f(x) = \text{Sin}x$ ve $f(x) = \text{Cose}$ çizelim.

* $f(x) = \text{Sin } x$

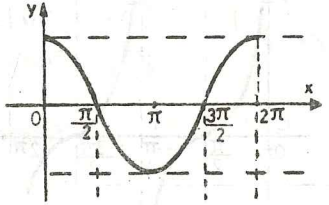


* $f(x) = \text{Cosec } x$

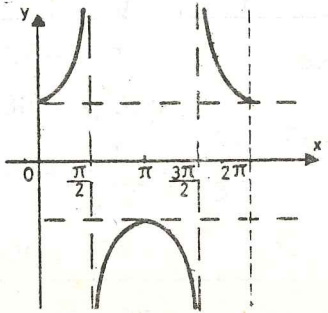


B) $0 \leq x \leq 2\pi$ için $f(x) = \text{Cos}x$ ve $f(x) = \text{Sec}x$ in grafiğini çizelim.

* $f(x) = \text{Cos}x$



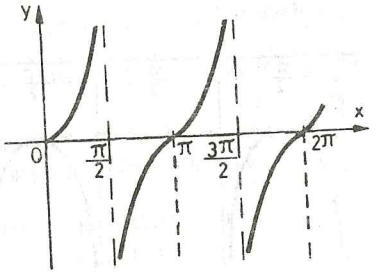
* $f(x) = \text{Sec}x$



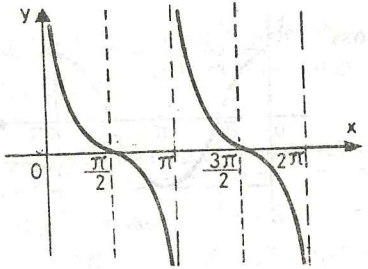
C) $0 \leq x \leq 2\pi$ için $f(x) = \tan x$ ve $f(x) = \text{Cotg}x$ in grafiklerini çizelim.

[$\tan x \cdot \text{Cotg}x = 1$ ve $\text{Cotg}x = \frac{1}{\text{tg}x}$ olduğunu hatırlayınız.]

* $f(x) = \tan x$

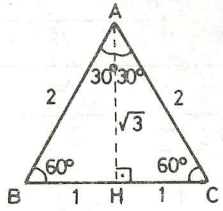


* $f(x) = \text{Cotg}\, x$



VI. 30°, 45°, 60° nin Trigonometrik Oranları :

a) 30° ve 60° için bir kenarı 2 birim olan bir eşkenar üçgen kullanırız. (Eşkenar üçgenin özelliklerini hatırlayınız.)



(A)

$\cos 60° = 1/2$ $\quad$ $\sec 60° = 2$

$\sin 60° = \sqrt{3}/2$ $\quad$ $\text{Cosec}60° = \frac{2}{\sqrt{3}}$

$\text{tg}\, 60° = \sqrt{3}$

$\text{Cotg}60° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

(B)

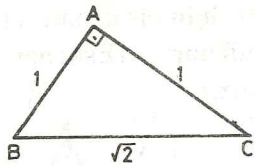
$\sin 30° = 1/2$ $\quad$ $\sec 30° = \frac{2}{\sqrt{3}}$

$\cos 30° = \sqrt{3}/2$ $\quad$ $\text{Cosec}30° = 2$

$\text{tg}\, 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\text{Cotg}30° = \sqrt{3}$

b) 45° için dik kenarları 1 birim olan ikizkenar dik üçgen kullanırız.



$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \sec 45^\circ = \sqrt{2}$

$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \operatorname{cosec} 45^\circ = \sqrt{2}$

$\operatorname{tg} 45^\circ = 1$

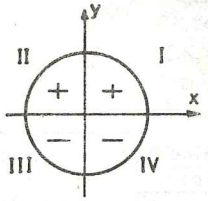
$\operatorname{cotg} 45^\circ = 1$

c) Bulduklarımızı bir tablo halinde özetleyelim.

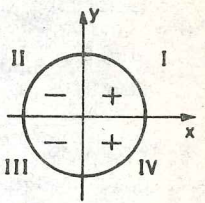
	Cos	Sin	Tan	Cotg
0°	1	0	0	Tanımsız
30°	$\sqrt{3}/2$	1/2	$\sqrt{3}/3$	$\sqrt{3}$
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1
60°	1/2	$\sqrt{3}/2$	$\sqrt{3}$	$\sqrt{3}/3$
90°	0	1	Tanımsız	0

VII. Bölgelerde İşaret İncelenmesi :

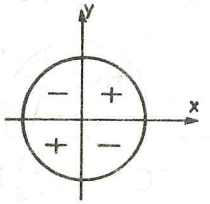
A) **Sinüs**



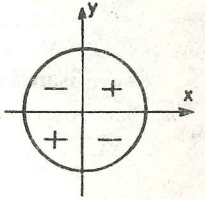
B) **Kosinüs**



C) **Tanjant**

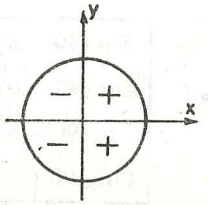


D) **Kotanjant**



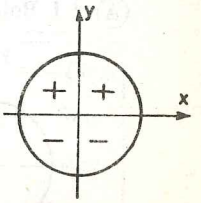
E) **Sekant**

(Kosinüs ile aynı işareti taşır)

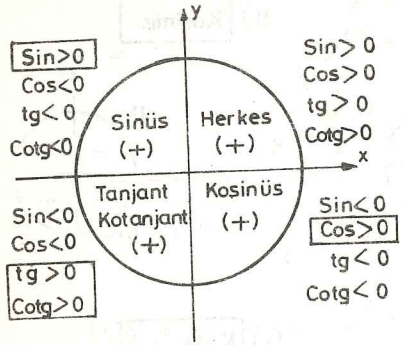


F) **Kosekant**

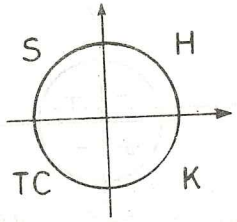
(Sinüs ile aynı işarette)



* Tek bir tabloda özetlersek,



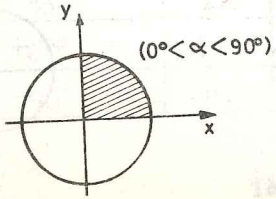
veya işareti (+) olanları kısaca özetleyelim.



[Herkes Sever T.C. Koşucularını]

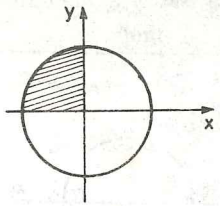
VIII. Bölgelerde Dönüşümler :

(A) I. Bölgede



$$\sin(90-\alpha) = \cos\alpha$$
$$\cos(90-\alpha) = \sin\alpha$$
$$\operatorname{tg}(90-\alpha) = \operatorname{Cotg}\alpha$$
$$\operatorname{Cotg}(90-\alpha) = \operatorname{tg}\alpha$$

(B) II. Bölgede

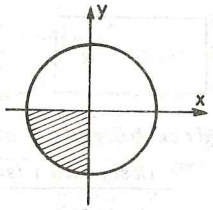


1)
$$\sin(180-\alpha) = \sin\alpha$$
$$\cos(180-\alpha) = -\cos\alpha$$
$$\operatorname{tg}(180-\alpha) = -\operatorname{tg}\alpha$$
$$\operatorname{Cotg}(180-\alpha) = -\operatorname{Cotg}\alpha$$

2)
$$\sin(90+\alpha) = \cos\alpha$$
$$\cos(90+\alpha) = -\sin\alpha$$
$$\operatorname{tg}(90+\alpha) = -\operatorname{Cotg}\alpha$$
$$\operatorname{Cotg}(90+\alpha) = -\operatorname{tg}\alpha$$

* II. bölgede önce işaret (+ veya –) bulunur. Sonra açı 180° ye tamamlanır.

(C) III. Bölgede

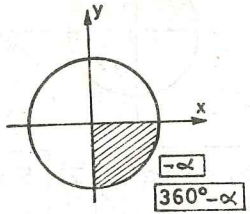


1)
$$\sin(180+\alpha) = -\sin\alpha$$
$$\cos(180+\alpha) = -\cos\alpha$$
$$\operatorname{tg}(180+\alpha) = \operatorname{tg}\alpha$$
$$\operatorname{Cotg}(180+\alpha) = \operatorname{Cotg}\alpha$$

2)
$$\sin(270-\alpha) = -\cos\alpha$$
$$\cos(270-\alpha) = -\sin\alpha$$
$$\operatorname{tg}(270-\alpha) = \operatorname{Cotg}\alpha$$
$$\operatorname{Cotg}(270-\alpha) = \operatorname{tg}\alpha$$

* III. bölgede önce işaret tayin edilir. Daha sonra 180° den fazlalık olan açı kullanılır.

Ⓓ IV. Bölgede



1) Sin (− α) = − Sin α
Cos (− α) = Cos α
tg (− α) = − tg α
Cotg(− α) = − Cotg α

2) Sin(270 + α) = − Cosα
Cos(270 + α) = Sin α
tg(270 + α) = − Cotgα
Cotg(270 + α) = − tg α

UYARI

1. $180 - \alpha$, $180 + \alpha$, $\frac{360 - \alpha}{-\alpha}$

Kalıplarında sadece bölgede işaret belirlenir ve α açısı kullanılır. Fonksiyonun ismi değişmez.

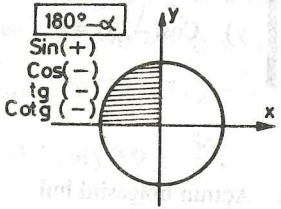
2. $90 - \alpha$, $90 + \alpha$, $270 - \alpha$, $270 + \alpha$

Kalıplarında bölgeye göre işaret belirlendikten sonra fonksiyonun ismi değiştirilir ve α açısı kullanılır.

ÖRNEK :

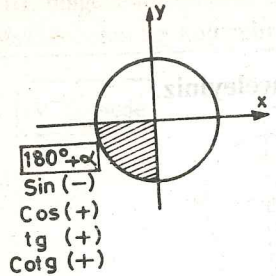
1. Aşağıdakileri dikkatle inceleyiniz.

a) Sin 100 = + Sin 80
b) Sin 130 = + Sin 50
c) Sin 140 = + Sin 40
d) Cos 120 = − Cos 60
e) Cos 140 = − Cos 40
f) tg 130 = − tg 50
g) Cotg170 = − Cotg 10



1) Açının bölgesini bul.
2) Bölgede işaret belirle.
3) İsmi değiştirme
4) 180° ye tamamlayan açıyı kullan

h) Sin 200 = − Sin 20
i) Cos 210 = − Cos 30
j) Cos 230 = − Cos 50
k) tg 240 = + tg 60
l) Cotg 250 = + Cotg 70

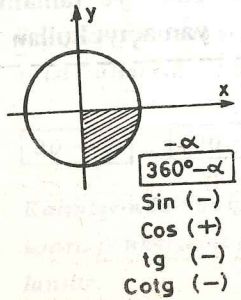


1) Açının bölgesini bul
2) Bölgede işaret incele
3) İsmi değiştirme
4) 180° den kaç derece fazla olduğunu bul ve bulduğun açıyı kullan

---

m) Cos 290 = + Cos 70
o) Sin 310 = − Sin 50
r) tg 330 = − tg 30
t) Cotg350 = − Cotg 10
v) Sin(−40) = − Sin 40
z) tg (− 45) = − tg 45

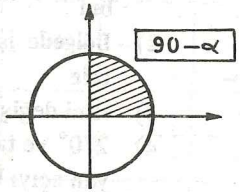
n) Cos 300 = + Cos 60
p) Sin 320 = − Sin 40
s) Cotg340 = − Cotg 20
u) Sin(−20) = − Sin 20
y) Cos(−25) = + Cos 25



1) Açının bölgesini bul
2) Bölgede işaret incele
3) İsmi değiştirme
4) a) 360° ye tamamlayan açıyı kullan
   b) − α şeklindeki açıları 360 − α şeklinde yaz ve işlem yap

2 Aşağıdakileri dikkatle inceleyiniz.

a) Sin (90 − 2 α) = Cos2 α
b) Cos(90 − 4 α) = Sin 4 α
c) tg (90 − 7 α) = Cotg 7 α
d) Cotg(90−6 α) = tg 6 α

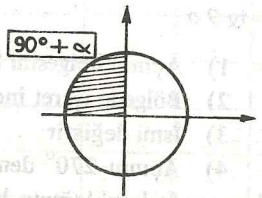


1) Açının bölgesini bul
2) Bölgede işaret incele
3) İsmi değiştir
4) 90° ye tamamlayan açıyı kullan

---

e) Sin (90 + 2 α) = Cos 2 α
f) Cos(90 + 3 α) = − Sin 3 α
g) tg (90 + 7 α) = − Cotg 7 α

h) $\text{Cotg}\left(90+\frac{3\alpha}{5}\right) = -\text{tg}\,\frac{3\alpha}{5}$



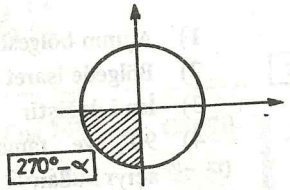
1) Açının bölgesini bul
2) Bölgede işaret incele
3) İsmi değiştir
4) 90° den kaç derece fazla olduğunu bul ve bulduğun açıyı kullan

i) $\text{Sin}\,(270 - 2\alpha) = -\,\text{Cos}\,2\alpha$

j) $\text{Cos}\,(270 - 3\alpha) = -\,\text{Sin}\,3\alpha$

k) $\text{tg}\,(270 - \frac{5\alpha}{7}) = +\,\text{Cotg}\,\frac{5\alpha}{7}$

l) $\text{Cotg}\,(270 - \frac{5\alpha}{3}) = +\,\text{tg}\,\frac{5\alpha}{3}$



1) Açının bölgesini bul
2) Bölgede işaret incele
3) İsmi değiştir
4) 270° ye tamamlayan açıyı kullan

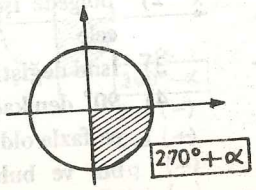
---

m) $\text{Sin}\,(270 + 5\alpha) = -\,\text{Cos}\,5\alpha$

n) $\text{Cos}\,(270 + \frac{7\alpha}{3}) = +\,\text{Sin}\,\frac{7\alpha}{3}$

o) $\text{tg}\,(270 + \frac{8\alpha}{17}) = -\,\text{Cotg}\,\frac{8\alpha}{17}$

p) $\text{Cotg}\,(270 + 9\alpha) = -\,\text{tg}\,9\alpha$



1) Açının bölgesini bul
2) Bölgede işaret incele
3) İsmi değiştir
4) Açının 270° den ne kadar fazla olduğunu bul ve bulduğun açıyı kullan

ALIŞTIRMALAR

1. Aşağıda verilen tabloyu doldurunuz. (Sadece işaretleri yazınız.)

Açı	Cos	Sin	tg	Cotg	Sec	Cosec
40°	+					
100°		+				
130°						
200°						
210°						
150°						
300°						
320°				—		
175°						
182°						
179°						
357°						
224°						
243°						
335°						
− 30°						
− 20°						
88°						

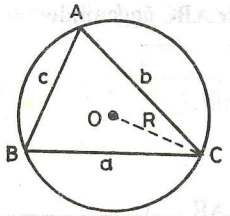
2. Aşağıdaki tabloyu doldurunuz.

Açı	Cos	Sin	tg	Cotg
30°				
45°				
60°				
120°				
135°				
150°				
210°				
225°				
240°				
300°				
315°				
330°				
0°				
90°				
180°				
270°				
360°				

5. BÖLÜM

SİNÜS – KOSİNÜS TEOREMLERİ TANJANT TEOREMİ

* **SİNÜS TEOREMİ :** Bir üçgende her kenar, karşısındaki açının sinüsü ile orantılıdır. Bu oranın sabitesi bu üçgenin çevrel çemberinin çapıdır.



$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC} = 2R$$

* **KOSİNÜS TEOREMİ :** Bir ABC üçgeninde ;

$$\left.\begin{aligned} a^2 &= b^2 + c^2 - 2\,bc\,CosA \\ b^2 &= a^2 + c^2 - 2\,ac\,CosB \\ c^2 &= a^2 + b^2 - 2\,ab\,Cos\,C \end{aligned}\right\}$$ bağıntıları vardır.

TANJANT TEOREMİ : Bir ABC üçgeninde ;

$$\frac{b+c}{b-c} = \frac{tg\left(\frac{B+C}{2}\right)}{tg\left(\frac{B-C}{2}\right)}$$ dir.

MOLVAYD FORMÜLLERİ : Bir ABC üçgeninde ;

$$\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{C}{2}\right)} \qquad \frac{a-b}{c} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{C}{2}\right)}$$

$$\frac{b+c}{a} = \frac{\cos\left(\frac{B-C}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad \frac{b-c}{a} = \frac{\sin\left(\frac{B-C}{2}\right)}{\cos\left(\frac{A}{2}\right)}$$

bağıntıları vardır.

İZDÜŞÜM FORMÜLLERİ : Bir ABC üçgeninde ;

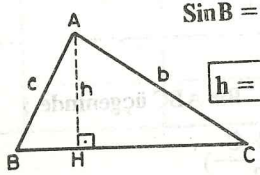
$$\left.\begin{aligned} b\,.\,\cos C + c\,.\,\cos B &= a \\ c\,.\,\cos A + a\,.\,\cos C &= b \\ a\,.\,\cos B + b\,.\,\cos A &= c \end{aligned}\right\} \text{ dir.}$$

ALIŞTIRMALAR

1. $\widehat{ABC}$ de sinüs teoremini ispatlayınız. (Dar açılı üçgende)

ÇÖZÜM :

a)



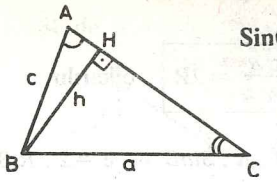
$\sin B = \frac{h}{c}$; $\sin C = \frac{h}{b}$

$\boxed{h = c\,.\,\sin B}$ $\wedge$ $\boxed{h = b\,.\,\sin C}$

$\Rightarrow$ c . SinB = b . SinC

buradan, $\boxed{\frac{b}{\sin B} = \frac{c}{\sin C}}$ bulunur.

b)



$\sin C = \frac{h}{a}$, $\sin A = \frac{h}{c}$

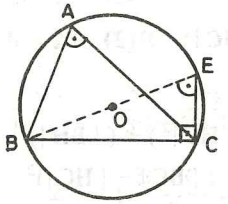
$\boxed{h = a\,.\,\sin C}$ $\wedge$ $\boxed{h = c\,.\,\sin A}$ $\Rightarrow$ a . SinC = c . SinA

buradan, $\boxed{\frac{a}{\sin A} = \frac{c}{\sin C}}$ bulunur.

(a) ve (b) birleştirilirse;

$$\boxed{\frac{a}{\sin A} = \frac{c}{\sin C} = \frac{b}{\sin B}}$$ çıkarılır.

c)



$\widehat{ABC}$ nin çevrel çemberini ve [BE] çapını çizelim.

$m\hat{C} = 90°$ dir. (Çapı gören çevre açı)

$m\hat{A} = m\hat{E}$ (Aynı yayı gören çevre açılar)

$\sin E = \frac{|BC|}{|BE|} = \frac{a}{2R} \Rightarrow \sin E = \frac{a}{2R}$ ve (SinE = SinA)

buradan ;

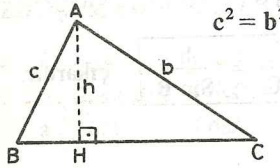
$\sin A = \frac{a}{2R}$ ve $\boxed{\frac{a}{\sin A} = 2R}$ bulunur.

(b) ve (c) birleştirilirse ;

$$\boxed{\frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}} = \frac{a}{\text{Sin A}} = 2R}$$ çıkarılır.

NOT : $a = 2 . R . SinA$, $b = 2 . R . SinB$ ve $c = 2 . R.SinC$ ye kiriş formülleri denir.

2. $\overset{\triangle}{ABC}$ de kosinüs teoremini ispatlayınız. (Dar açılı üçgende)



$c^2 = b^2 + a^2 - 2ab\ CosC$ yi ispatlayalim.

$\overset{\triangle}{ABH}$ de $|AB|^2 = |AH|^2 + |BH|^2$ (1)

$\overset{\triangle}{AHC}$ de $|AH|^2 = |AC|^2 - |HC|^2$ (2)

(2) yi (1) de yerine koyalım.

$$|AB|^2 = (|AC|^2 - |HC|^2) + (|BH|)^2$$

$$c^2 = (b^2 - |HC|^2) + (|BC| - |HC|)^2$$

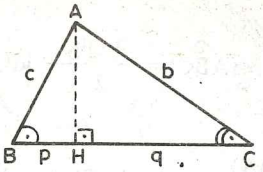
$$c^2 = (b^2 - \cancel{|HC|^2}) + (|BC|^2 - 2|BC| \cdot |CH| + \cancel{|HC|^2})$$

$c^2 = b^2 + a^2 - 2 . a . |CH|$ bulunur. (3)

$\overset{\triangle}{AHC}$ de $CosC = \dfrac{|HC|}{b}$ ve $|HC| = b . Cos\ C$ dir. (3) de yerine koyalım.

$$c^2 = b^2 + a^2 - 2a . b . CosC \text{ bulunur.}$$

3. $\overset{\triangle}{ABC}$ de $b . CosC + c . CosB = a$ olduğunu gösteriniz.



$$b . CosC + c . CosB = b\left(\frac{q}{b}\right) + c\left(\frac{p}{c}\right) = q + p = a \text{ dir.}$$

$$\boxed{CosB = \frac{p}{c}} \qquad \boxed{CosC = \frac{q}{b}}$$

4. $\overset{\triangle}{ABC}$ de $\dfrac{b+c}{b-c} = \dfrac{tg\left(\frac{B+C}{2}\right)}{tg\left(\frac{B-C}{2}\right)}$ (tanjant teoremi) olduğunu gösteriniz.

$\dfrac{b}{\text{Sin B}} = \dfrac{c}{\text{Sin C}} = 2R$ idi. (Sinüs teoremi) Buradan;

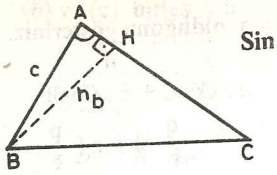
$b = 2\ R\ Sin\ B$ ve $c = 2\ R\ SinC$ bulunur.

$$\frac{b+c}{b-c} = \frac{2R(SinB + SinC)}{2R(SinB - SinC)} = \frac{SinB + SinC}{SinB - SinC}$$

$$= \frac{\cancel{2}Sin\left(\frac{B+C}{2}\right) . Cos\left(\frac{B-C}{2}\right)}{\cancel{2}Sin\left(\frac{B-C}{2}\right) . Cos\left(\frac{B+C}{2}\right)} = \frac{tg\left(\frac{B+C}{2}\right)}{tg\left(\frac{B-C}{2}\right)}$$

dir.

5. $\overset{\triangle}{ABC}$ de $A(\overset{\triangle}{ABC}) = \dfrac{1}{2} \cdot b . c . SinA$ olduğunu gösteriniz.

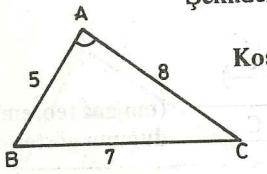


$\text{Sin A} = \dfrac{h_b}{c}$ dir. $h_b = c \,.\, \text{SinA}$ olur.

$(A(\widehat{ABC}) = \dfrac{b \,.\, h_b}{2}$ idi.)

$A(\widehat{ABC}) = \dfrac{b \,.\, (c \,.\, \text{SinA})}{2} = \dfrac{1}{2}$ bc SinA olur.

6. Şekildeki $\widehat{ABC}$ de $m\hat{A} = ?$



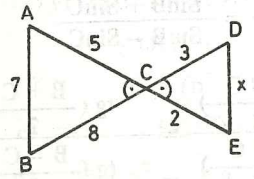
Kosinüs teoreminden;

$a^2 = b^2 + c^2 - 2\,bc\,\text{CosA}$ dır.

$\boxed{\text{CosA} = \dfrac{b^2 + c^2 - a^2}{2bc}}$ dir.

$\text{CosA} = \dfrac{8^2 + 5^2 - 7^2}{2 \,.\, 5 \,.\, 8} = \dfrac{40}{80} = \dfrac{1}{2} \Rightarrow \boxed{m\hat{A} = 60°}$ dir.

7. Şekilde x kaçtır ?



* Kosinüs teoremini iki kere uygulayalım. $\widehat{ABC}$ de;

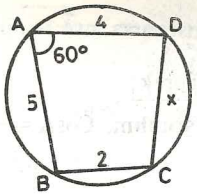
$\text{CosC} = \dfrac{5^2 + 8^2 - 7^2}{2 \,.\, 5 \,.\, 8} = \dfrac{1}{2}$ ve

$\widehat{CDE}$ de ; $x^2 = 2^2 + 3^2 - 2 \,.\, 2 \,.\, 3 \,.\, \text{CosC}$

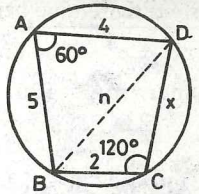
$x^2 = 4 + 9 - 6$

$\boxed{x = \sqrt{7}}$ dir.

8.



Şekilde ABCD kirişler dörtgenidir. x kaçtır ?



(Kirişler dörtgeninde, $m\hat{A} + m\hat{C} = 180°$)

$\widehat{ABD}$ de $n^2 = 4^2 + 5^2 - 2 \,.\, 4 \,.\, 5 \,.\, \text{Cos60}$

$\boxed{n^2 = 21}$ dir.

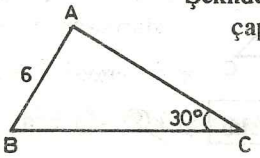
$\widehat{BDC}$ de $n^2 = 2^2 + x^2 - 2 \,.\, 2 \,.\, x \,.\, \text{Cos}(120)$

$21 = 4 + x^2 + 2x$

$x^2 + 2x - 17 = 0$

$\boxed{x = 3\sqrt{2} - 1}$ dir.

9. Şekildeki $\widehat{ABC}$ nin çevrel çemberinin yarıçapı nedir ?



$\dfrac{c}{\text{Sin C}} = 2R$, (Sinüs teoremi)

$R = \dfrac{c}{2\text{SinC}} = \dfrac{6}{2\text{Sin30}} = \dfrac{6}{1} = 6$ dır.

10. $\triangle ABC$ de $a^2 = b^2 + c^2 + bc$ bağıntısı varsa, $m\hat{A}$ nedir ?

Kosinüs teoremine benzediği düşünülerek;

$a^2 = b^2 + c^2 - 2bc\,CosA$ ile karşılaştıralım. $CosA = -\frac{1}{2}$ bulunur. $m\hat{A} = 120°$ dir.

11. Bir $\triangle ABC$ de $b^3 + c^3 - a^3 = a^2\,(b + c - a)$ ise, $m\hat{A}$ nedir ?

İfadeyi yeniden düzenleyelim.

$b^3 + c^3 - a^3 = a^2 b + a^2 c - a^3$; [$b^3 + c^3 = (b + c)(b^2 - bc + c^2$ olduğunu hatırlayınız.]

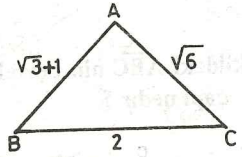
$(b + c)(b^2 - bc + c^2) = a^2\,(b + c)$

$\boxed{a^2 = b^2 + c^2 - bc}$ çıkar.

(10. soru gibi düşünürsek)

$CosA = \frac{1}{2}$ bulunur. $\Rightarrow m\hat{A} = 60°$ dir.

12. $\triangle ABC$ de $a = 2$, $b = \sqrt{6}$ ve $c = \sqrt{3} + 1$ ise, $m\hat{C}$ nedir?



a) $\boxed{a^2 = b^2 + c^2 - 2bc\,CosA \text{ idi}}$

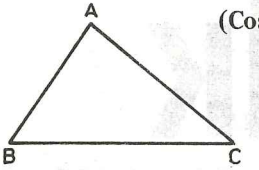
$$CosA = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{\sqrt{2}} \Rightarrow m\hat{A} = 45°$$

b) $\boxed{b^2 = a^2 + c^2 - 2ac\,CosB \text{ idi}}$

$$CosB = \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2} \Rightarrow m\hat{B} = 60°$$

buradan $m\hat{C} = 75°$ bulunur.

13. $\triangle ABC$ de $a = 5$, $b = 3$ ve $CosC = -\frac{4}{5}$ ise, bu üçgenin çevrel çemberinin yarıçapı nedir ?



($CosC = -\frac{4}{5}$ ise, C geniş açı ve $SinC = \frac{3}{5}$ dir.)

$c^2 = a^2 + b^2 - 2ab\,CosC$ idi;

$c^2 = 5^2 + 3^2 - 2 \cdot 5 \cdot 3 \cdot (-\frac{4}{5}) \Rightarrow c = \sqrt{58}$ dir.

$\frac{c}{Sin\,C} = 2R$ den $\frac{\sqrt{58}}{2 \cdot \frac{3}{5}} = R \Rightarrow R = \frac{5\sqrt{58}}{6}$ dır.

14. $\triangle ABC$ de $b = 2 \cdot c \cdot CosA$ ise, bu üçgenin ikizkenar üçgen olduğunu gösteriniz.

Kosinüs Teoreminden,

$a^2 = b^2 + c^2 - \underbrace{2\,b\,c\,CosA}_{b}$ dir. $\Rightarrow a^2 = b^2 + c^2 - \underbrace{b(2c\,CosA)}_{b}$

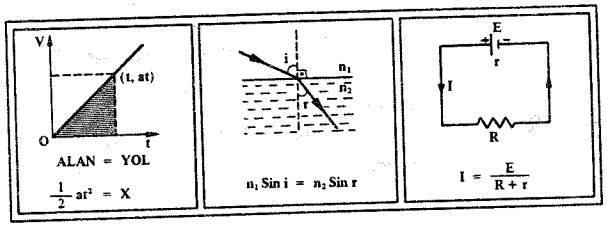
$\Rightarrow a^2 = b^2 + c^2 - b^2 \Rightarrow a^2 = c^2 \Rightarrow \boxed{a = c}$ bulunur.

büyük
yayınları

YENİ SINAV SİSTEMİNE GÖRE HAZIRLANMIŞTIR

ÖYS
İÇİN

FİZİK



• ÖYS konularının detaylı açıklaması
• 769 Çözümlü örnek test
• 940 Cevaplı test

Nurettin Ceran Necati Tanrıöver

6. BÖLÜM

İKİ AÇININ TOPLAM VE FARKININ TRİGONOMETRİK FONKSİYONLARI

TOPLAM FORMÜLLERİ

(6-1) $\text{Sin}(\alpha + \beta) = \text{Sin}\,\alpha \,.\, \text{Cos}\,\beta + \text{Cos}\,\alpha \,.\, \text{Sin}\,\beta$

(6-2) $\text{Cos}(\alpha + \beta) = \text{Cos}\,\alpha \,.\, \text{Cos}\,\beta - \text{Sin}\,\alpha \,.\, \text{Sin}\,\beta$

(6-3) $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \,.\, \tan\beta}$

(6-4) $\text{Cotg}(\alpha + \beta) = \dfrac{\text{Cotg}\,\alpha \,.\, \text{Cotg}\,\beta - 1}{\text{Cotg}\,\alpha + \text{Cotg}\,\beta}$

FARK FORMÜLLERİ

(6-5) $\text{Sin}(\alpha - \beta) = \text{Sin}\,\alpha \,.\, \text{Cos}\,\beta - \text{Cos}\,\alpha\,\text{Sin}\,\beta$

(6-6) $\text{Cos}(\alpha - \beta) = \text{Cos}\,\alpha \,.\, \text{Cos}\,\beta + \text{Sin}\,\alpha\,\text{Sin}\,\beta$

(6-7) $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha \,.\, \tan\beta}$

(6-8) $\text{Cotg}(\alpha - \beta) = \dfrac{\text{Cotg}\,\alpha \,.\, \text{Cotg}\,\beta + 1}{\text{Cot}\,\beta - \text{Cotg}\,\alpha}$

YARIM AÇI FORMÜLLERİ

(6-9) $\text{Sin}\,2\alpha = 2\,\text{Sin}\,\alpha \,.\, \text{Cos}\,\alpha$

(6-10) $\text{Cos}\,2\alpha = \text{Cos}^2\alpha - \text{Sin}^2\alpha$ (a)

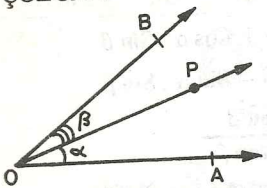
$= 1 - 2\,\text{Sin}^2\alpha$ (b)

$= 2\,\text{Cos}^2\alpha - 1$ (c)

(6-11) $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

1. $\text{Sin}(\alpha+\beta) = \text{Sin}\,\alpha \,.\, \text{Cos}\,\beta + \text{Cos}\,\alpha \,.\, \text{Sin}\,\beta$ olduğunu gösteriniz.

ÇÖZÜM :



Şekil 6/1

Birer kenarları ortak $\widehat{AOP}$ ve $\widehat{PO}$ açılarını alalım. Ortak olan kenar üzerinde bir P noktası seçelim.

Seçilen bu P noktasından | OP ye bir dik çizelim.

$$(1)\ A(\widehat{AOB}) = \frac{1}{2} \cdot |OA| \cdot |OB| \,.\, \text{Sin}(\alpha+\beta)$$

$$(2)\ A(\widehat{OAP}) = \frac{1}{2} \cdot |OA| \cdot |OP| \,.\, \text{Sin}\,\alpha$$

$$(3)\ A(\widehat{OPB}) = \frac{1}{2} \cdot |OP| \cdot |OB| \,.\, \text{Sin}\,\beta$$

$A(\widehat{AOB}) = A(\widehat{OAP}) + A(\widehat{OPB})$ dir. Buradan

$$\frac{1}{2}|OA| \,.\, |OB| \,.\, \text{Sin}(\alpha+\beta) = \frac{1}{2}|OA| \,.\, |OP| \,.\, \text{Sin}\,\alpha + \frac{1}{2}|OP| \,.\, |OB| \,.\, \text{Sin}\,\beta$$

$$\text{Sin}(\alpha+\beta) = \frac{|OA| \,.\, |OP| \,.\, \text{Sin}\,\alpha}{|OA| \,.\, |OB|} + \frac{|OP| \,.\, |OB| \,.\, \text{Sin}\beta}{|OA| \,.\, |OB|}$$

[Her iki tarafı $\text{Sin}(\alpha+\beta)$ nın katsayısı ile böldük.]

$$\text{Sin}(\alpha+\beta) = \frac{|OP|}{|OB|} \,.\, \text{Sin}\,\alpha + \frac{|OP|}{|OA|} \,.\, \text{Sin}\,\beta$$

OPB dik üçgeninde $\text{Cos}\,\beta = \dfrac{|OP|}{|OB|}$

OPA dik üçgeninde $\text{Cos}\,\alpha = \dfrac{|OP|}{|OA|}$ olduğunu görebiliriz.

$$\text{Sin}(\alpha+\beta) = \text{Cos}\,\beta \,.\, \text{Sin}\,\alpha + \text{Cos}\,\alpha \,.\, \text{Sin}\,\beta$$
$$= \text{Sin}\,\alpha \,.\, \text{Cos}\,\beta + \text{Cos}\,\alpha \,.\, \text{Sin}\,\beta \text{ dir.}$$

2. $\text{Cos}(\alpha+\beta) = \text{Cos}\,\alpha \,.\, \text{Cos}\,\beta - \text{Sin}\,\alpha \,.\, \text{Sin}\,\beta$ olduğunu gösteriniz.

ÇÖZÜM : (6 - 2) de Kosinüs Teoremini kullanırsak;

$|AB|^2 = |OA|^2 + |OB|^2 - 2 \,.\, |OA| \,.\, |OB| \,.\, \text{Cos}(\alpha+\beta)$ buradan

$$2|OA| \,.\, |OB| \,.\, \text{Cos}(\alpha+\beta) = |OA|^2 + |OB|^2 - |AB|^2$$

[Her iki tarafı $\text{Cos}(\alpha+\beta)$ nın katsayısı ile bölelim.]

$$\text{Cos}(\alpha+\beta) = \frac{|OA|^2 + |OB|^2 - |AB|^2}{2|OA| \,.\, |OB|}$$

OPB dik üçgeninde $|OP|^2 + |PB|^2 = |OB|^2$

OAP dik üçgeninde $|OP|^2 + |AP|^2 = |OA|^2$ olduğunu Pisagor Teoremini kullanarak bulabiliriz.

$$\text{Cos}(\alpha+\beta) = \frac{|OP|^2 + |AP|^2 + |OP|^2 + |PB|^2 - |(AP+PB)|^2}{2|OA| \,.\, |OB|}$$

[$|AB| = |AP| + |PB|$ olduğunu görünüz.]

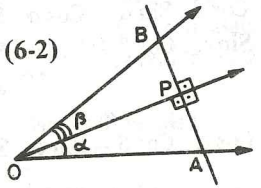
$$\text{Cos}(\alpha+\beta) = \frac{2|OP|^2 + |AP|^2 + |PB|^2 - [\,|AP|^2 + 2\,|AP|.|PB| + |PB|\,]^2}{2|OA| \,.\, |OB|}$$

$$\text{Cos}(\alpha+\beta)=\frac{2\,|OP|^2+|AP|^2+|PB|^2-2\,|AP|\,.\,|PB|-|PB|^2-|}{2\,|OA|\,.\,|OB|}$$

$$\text{Cos}(\alpha+\beta)=\frac{2|OP|^2}{2|OA|\,.\,|OB|}-\frac{2|AP|\,.\,|PB|}{2|OA|\,.\,|OB|}$$

$$\text{Cos}(\alpha+\beta)=\frac{|OP|}{|OA|}\cdot\frac{|OP|}{|OB|}-\frac{|AP|}{|OA|}\cdot\frac{|PB|}{|OB|}$$

Şekil (6-2)



Şekil 6 - 2 de

$$\text{Sin}\,\alpha=\frac{|AP|}{|OA|}\qquad \text{Sin}\,\beta=\frac{|PB|}{|OB|}$$

$$\text{Cos}\,\alpha=\frac{|OP|}{|OA|}\qquad \text{Cos}\,\beta=\frac{|OP|}{|OB|}$$ olduğunu görünüz.

$\text{Cos}(\alpha+\beta)=\text{Cos}\,\alpha\,.\,\text{Cos}\,\beta-\text{Sin}\,\alpha\,.\,\text{Sin}\,\beta$ olduğunu buluruz.

3. $\text{tg}(\alpha+\beta)=\dfrac{\text{tg}\,\alpha+\text{tg}\,\beta}{1-\text{tg}\,\alpha\,.\,\text{tg}\,\beta}$ olduğunu gösteriniz.

ÇÖZÜM :

$$\text{tg}(\alpha+\beta)=\frac{\text{Sin}(\alpha+\beta)}{\text{Cos}(\alpha+\beta)}$$ [$\text{tg}\,\theta=\dfrac{\text{Sin}\,\theta}{\text{Cos}\,\theta}$ olduğunu

layarak daha önce bulduğ

açılımları kullanalım.]

$$\text{tg}(\alpha+\beta)=\frac{\text{Sin}\,\alpha\,.\,\text{Cos}\,\beta+\text{Cos}\,\alpha\,.\,\text{Sin}\,\beta}{\text{Cos}\,\alpha\,.\,\text{Cos}\,\beta-\text{Sin}\,\alpha\,.\,\text{Sin}\,\beta}$$ [Pay ve paydayı, $\text{Cos}\,\alpha\,.\,\text{Cos}\,\beta$ ile bölelim.]

$$\text{tg}(\alpha+\beta)=\frac{\dfrac{\text{Sin}\,\alpha\,.\,\cancel{\text{Cos}\,\beta}}{\text{Cos}\,\alpha\,.\,\cancel{\text{Cos}\,\beta}}+\dfrac{\cancel{\text{Cos}\,\alpha}\,.\,\text{Sin}\,\beta}{\cancel{\text{Cos}\,\alpha}\,.\,\text{Cos}\,\beta}}{\dfrac{\cancel{\text{Cos}\,\alpha\,.\,\text{Cos}\,\beta}}{\cancel{\text{Cos}\,\alpha\,.\,\text{Cos}\,\beta}}-\dfrac{\text{Sin}\,\alpha\,.\,\text{Sin}\,\beta}{\text{Cos}\,\alpha\,.\,\text{Cos}\,\beta}}$$

$$=\frac{\dfrac{\text{Sin}\,\alpha}{\text{Cos}\,\alpha}+\dfrac{\text{Sin}\,\beta}{\text{Cos}\,\beta}}{1-\dfrac{\text{Sin}\,\alpha}{\text{Cos}\alpha}\cdot\dfrac{\text{Sin}\,\beta}{\text{Cos}\,\beta}}$$

$\text{tg}(\alpha+\beta)=\dfrac{\text{tg}\,\alpha+\text{tg}\,\beta}{1-\text{tg}\,\alpha\,.\,\text{tg}\,\beta}$ olduğunu buluruz.

4. $\text{Sin}(\alpha-\beta)=\text{Sin}\,\alpha\,.\,\text{Cos}\,\beta-\text{Cos}\,\alpha\,.\,\text{Sin}\,\beta$ olduğunu gösteriniz.

ÇÖZÜM : I. Yol :

$\text{Sin}(\alpha+\beta)=\text{Sin}\,\alpha\,.\,\text{Cos}\,\beta+\text{Cos}\,\alpha\,.\,\text{Sin}\,\beta$ olduğunu bulmuştuk.

β yerine $-\beta$ koyarsak;

$\text{Sin}(\alpha+(-\beta))=\text{Sin}\alpha\,.\,\text{Cos}(-\beta)+\text{Cos}\alpha\,.\,\text{Sin}(-\beta)$

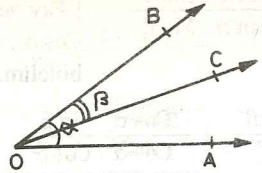
$\text{Sin}(\alpha-\beta)=\text{Sin}\,\alpha\,.\,\text{Cos}\,\beta+\text{Cos}\,\alpha\cdot[-\text{Sin}\,\beta]$

$\text{Sin}(\alpha-\beta)=\text{Sin}\,\alpha\,.\,\text{Cos}\,\beta-\text{Cos}\,\alpha\,.\,\text{Sin}\,\beta$ bulunur.

($\text{Cos}(-\theta)=\text{Cos}\,\theta$
$\text{Sin}(-\theta)=-\text{Sin}\,\theta$ olduğunu hatırlayınız.)

II. Yol :

Şekil 6 - 3



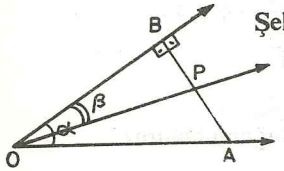
$m(\widehat{AOB}) = \alpha$ ve

$m(\widehat{COB}) = \beta$ alırsak,

$m(\widehat{AOC}) = \alpha - \beta$ olur.

B noktasından [OB ye dik indirirsek;

Şekil 6 - 4



$A(\triangle AOB) = A(\triangle OAP) + A(\triangle OPB)$ ve

$A(\triangle AOP) = A(\triangle AOB) - A(\triangle OPB)$

bulunur.

$\frac{1}{2} \cdot |OA| \cdot |OP| \cdot \text{Sin}(\alpha - \beta)$

$= \frac{1}{2} |OA| \cdot |OB| \cdot \text{Sin}\,\alpha - \frac{1}{2} \cdot |OP| \cdot |OB| \cdot \text{Sin}\,\beta$

$\text{Sin}(\alpha - \beta) = \frac{|OB|}{|OP|} \cdot \text{Sin}\,\alpha - \frac{|OB|}{|OA|} \cdot \text{Sin}\,\beta$ olur.

OPB dik üçgeninde $\text{Cos}\,\beta = \frac{|OB|}{|OP|}$

OAB dik üçgeninde $\text{Cos}\,\alpha = \frac{|OB|}{|OA|}$ olduğunu görebilirsiniz.

$\text{Sin}(\alpha - \beta) = \text{Cos}\,\beta \cdot \text{Sin}\,\alpha - \text{Cos}\,\alpha \cdot \text{Sin}\,\beta$

$\text{Sin}(\alpha - \beta) = \text{Sin}\,\alpha \cdot \text{Cos}\,\beta - \text{Cos}\,\alpha \cdot \text{Sin}\,\beta$ olduğunu bulabiliriz.

5. $\text{Cos}(\alpha - \beta) = \text{Cos}\,\alpha \cdot \text{Cos}\,\beta + \text{Sin}\,\alpha \cdot \text{Sin}\,\beta$ olduğunu gösteriniz.

ÇÖZÜM : I. Yol ,

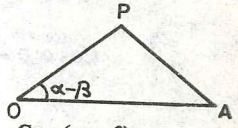
$\text{Cos}(\alpha + \beta) = \text{Cos}\,\alpha \cdot \text{Cos}\,\beta - \text{Sin}\,\alpha \cdot \text{Sin}\,\alpha \cdot \text{Sin}\,\beta$ olduğunu bulmuştuk.

β yerine $-\beta$ koyarsak;

$\text{Cos}(\alpha + (-\beta)) = \text{Cos}\,\alpha \cdot \text{Cos}(-\beta) - \text{Sin}\,\alpha \cdot \text{Sin}(-\beta)$

$\text{Cos}(\alpha - \beta) = \text{Cos}\,\alpha \cdot \text{Cos}\,\beta + \text{Sin}\,\alpha \cdot \text{Sin}\,\beta$ olur.

II. Yol : Şekil 6-4 de



$\widehat{AOP}$ de kosinüs teoremini uygulayalım.

$|AP|^2 = |OA|^2 + |OP|^2 - 2\,|OA| \cdot |OP| \cdot \text{Cos}(\alpha - \beta)$

[$m(\widehat{AOP}) = \alpha - \beta$ idi.]

$$\text{Cos}(\alpha - \beta) = \frac{|OA|^2 + |OP|^2 - |AP|^2}{2\,|OA| \cdot |OP|}$$

$$= \frac{|OA|^2 + |OP|^2 - [\,|AB| - |PB|\,]^2}{2\,|OA| \cdot |OP|}$$

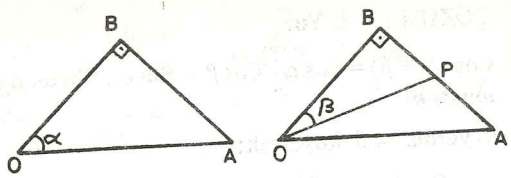
$$\left.\begin{aligned} |AP| &= |AB| - |PB| \\ |OA|^2 &= |OB|^2 + |AB|^2 \\ |OP|^2 &= |OB|^2 + |BP|^2 \end{aligned}\right\} \text{ dir.}$$

$$\text{Cos}(\alpha - \beta) = \frac{|OB|^2 + \cancel{|AB|^2} + |OB|^2 + \cancel{|BP|^2} - \cancel{|AB|^2} + 2\,|AB| \cdot |PB| - \cancel{|PB|^2}}{2\,|OA| \cdot |OP|}$$

$$\text{Cos}(\alpha - \beta) = \frac{2\,|OB|^2}{2\,|OA| \cdot |OP|} + \frac{2\,|AB| \cdot |PB|}{2\,|OA| \cdot |OP|}$$

$$= \frac{|OB|}{|OA|} \cdot \frac{|OB|}{|OP|} + \frac{|AB|}{|OA|} \cdot \frac{|PB|}{|OP|}$$

Şekil 6 - 4'ü parçalarsak;



$\text{Sin}\,\alpha = \frac{|AB|}{|OA|}$ $\quad$ $\text{Sin}\,\beta = \frac{|BP|}{|OP|}$

$\text{Cos}\,\alpha = \frac{|OB|}{|OA|}$ $\quad$ $\text{Cos}\,\beta = \frac{|OB|}{|OP|}$ dir.

$\text{Cos}\,(\alpha - \beta) = \text{Cos}\,\alpha \,.\, \text{Cos}\,\beta + \text{Sin}\,\alpha \,.\, \text{Sin}\,\beta$ bulunur.

6. $\text{Sin}\,2\alpha = 2\,\text{Sin}\,\alpha \,.\, \text{Cos}\,\alpha$ olduğunu gösteriniz.

ÇÖZÜM : $\text{Sin}\,(\alpha + \beta) = \text{Sin}\,\alpha \,.\, \text{Cos}\,\beta + \text{Cos}\,\alpha \,.\, \text{Sin}\,\beta$ idi.

β yerine α yazarsak ;

$\text{Sin}\,(\alpha + \alpha) = \text{Sin}\,\alpha \,.\, \text{Cos}\,\alpha + \text{Cos}\,\alpha \,.\, \text{Sin}\,\alpha$

$= 2\,\text{Sin}\,\alpha \,.\, \text{Cos}\,\alpha$ olur.

7. $\text{Cos}\,2\alpha = \text{Cos}^2\,\alpha - \text{Sin}^2\,\alpha = 2\,\text{Cos}^2\,\alpha - 1 = 1 - 2\,\text{Sin}^2\,\alpha$ oldu-
ğunu gösteriniz.

ÇÖZÜM : $\text{Cos}\,(\alpha + \beta) = \text{Cos}\,\alpha \,.\, \text{Cos}\,\beta - \text{Sin}\,\alpha \,.\, \text{Sin}\,\beta$ idi.

β yerine α yazarsak;

$\text{Cos}\,(\alpha + \alpha) = \text{Cos}\,\alpha \,.\, \text{Cos}\,\alpha - \text{Sin}\,\alpha \,.\, \text{Sin}\,\alpha$

$\text{Cos}^2\,\alpha = \text{Cos}^2\,\alpha - \text{Sin}^2\,\alpha$ olur.

[$\text{Cos}^2\,\alpha = 1 - \text{Sin}^2\,\alpha$ ve $\text{Sin}^2\,\alpha = 1 - \text{Cos}^2\,\alpha$ idi.]

a) $\text{Cos}2\alpha = (1 - \text{Sin}^2\,\alpha) - \text{Sin}^2\,\alpha = 1 - 2\,\text{Sin}^2\,\alpha$

b) $\text{Cos}2\alpha = \text{Cos}^2\,\alpha - (1 - \text{Cos}^2\,\alpha) = \text{Cos}^2\,\alpha - 1 + \text{Cos}^2\,\alpha$
$= 2\,\text{Cos}^2\,\alpha - 1$ olur.

8. $\text{tg}\,2\alpha = \frac{2\,\text{tg}\,\alpha}{1 - \text{tg}^2\,\alpha}$ olduğunu gösteriniz.

ÇÖZÜM :

$\text{tg}\,(\alpha + \beta) = \frac{\text{tg}\,\alpha + \text{tg}\,\beta}{1 - \text{tg}\,\alpha \,.\, \text{tg}\,\beta}$ idi. β yerine α yazarsak;

$\text{tg}\,(\alpha + \beta) = \frac{\text{tg}\,\alpha + \text{tg}\,\alpha}{1 - \text{tg}\,\alpha \,.\, \text{tg}\,\alpha} = \frac{2\,\text{tg}\,\alpha}{1 - \text{tg}^2\,\alpha}$ olur.

9. $\text{Sin}\,3\alpha = 3\,\text{Sin}\,\alpha - 4\,\text{Sin}^3\,\alpha$ olduğunu gösteriniz. ($\text{Sin}\,\alpha$ ya bağlı yazınız.)

ÇÖZÜM : $3\alpha = 2\alpha + \alpha$ şeklinde yazılırsa;

$\text{Sin}\,(3\alpha) = \text{Sin}\,(2\alpha + \alpha) = \text{Sin}\,2\alpha \,.\, \text{Cos}\,\alpha + \text{Cos}2\alpha \,.\, \text{Sin}\,\alpha$

$\text{Sin}\,2\alpha = 2\,\text{Sin}\,\alpha \,.\, \text{Cos}\,\alpha$
$\text{Cos}\,2\alpha = 1 - 2\,\text{Sin}^2\,\alpha$ idi.

$\text{Sin}3\alpha = (2\,\text{Sin}\,\alpha \,.\, \text{Cos}\,\alpha) \,.\, \text{Cos}\,\alpha + (1 - 2\,\text{Sin}^2\alpha) \,.\, \text{Sin}\,\alpha$
$= 2\,\text{Sin}\,\alpha \,.\, (\text{Cos}^2\,\alpha) + \text{Sin}\,\alpha - 2\,\text{Sin}^3\,\alpha$
[$\text{Cos}^2\,\alpha = 1 - \text{Sin}^2\,\alpha$ idi.]

$\text{Sin}3\alpha = 2\,\text{Sin}\,\alpha\,(1 - \text{Sin}^2\,\alpha) + \text{Sin}\,\alpha - 2\,\text{Sin}^3\,\alpha$

$\text{Sin}3\alpha = 2\,\text{Sin}\,\alpha - 2\,\text{Sin}^3\,\alpha + \text{Sin}\,\alpha - 2\,\text{Sin}^3\,\alpha$

$\text{Sin}3\alpha = 3\,\text{Sin}\,\alpha - 4\,\text{Sin}^3\,\alpha$ olur.

10 $\text{Cos}3\alpha = 4\,\text{Cos}^3\alpha - 3\,\text{Cos}\alpha$ olduğunu gösteriniz.

ÇÖZÜM : $3\alpha = 2\alpha + \alpha$ şeklinde yazılırsa;

$\text{Cos}(3\alpha) = \text{Cos}(2\alpha + \alpha) = \text{Cos}2\alpha \,.\, \text{Cos}\alpha - \text{Sin}2\alpha \,.\, \text{Sin}\alpha$

$\text{Cos}2\alpha = 2\,\text{Cos}^2\alpha - 1$

$\text{Sin}2\alpha = 2\,\text{Sin}\alpha \,.\, \text{Cos}\alpha$ idi.

$\text{Cos}3\alpha = (2\,\text{Cos}^2\alpha - 1) \,.\, \text{Cos}\alpha - (2\,\text{Sin}\alpha \,.\, \text{Cos}\alpha) \,.\, \text{Sin}\alpha$

$= 2\,\text{Cos}^3\alpha - \text{Cos}\alpha - 2\,(\text{Sin}^2\alpha) \,.\, \text{Cos}\alpha$

$[\,\text{Sin}^2\alpha = 1 - \text{Cos}^2\alpha$ idi. $]$

$\text{Cos}3\alpha = 2\,\text{Cos}^3\alpha - \text{Cos}\alpha - 2\,(1 - \text{Cos}^2\alpha) \,.\, \text{Cos}\alpha$

$= 2\,\text{Cos}^3\alpha - \text{Cos}\alpha - 2\,\text{Cos}\alpha + 2\,\text{Cos}^3\alpha$

$\text{Cos}3\alpha = 4\,\text{Cos}^3\alpha - 3\,\text{Cos}\alpha$ bulunur.

11 $\text{Sin}15^\circ, \text{Cos}15^\circ, \text{tg}15^\circ$ in değerini hesaplayınız.

ÇÖZÜM : $15^\circ = 45^\circ - 30^\circ$ veya $15^\circ = 60^\circ - 45^\circ$ olarak yazılabilir.

a) $\text{Sin}15 = \text{Sin}(45^\circ - 30^\circ) = \text{Sin}45 \,.\, \text{Cos}30 - \text{Cos}45 \,.\, \text{Sin}30$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

VEYA

$\text{Sin}15^\circ = \text{Sin}(60^\circ - 45^\circ) = \text{Sin}60 \,.\, \text{Cos}45 - \text{Cos}60 \,.\, \text{Sin}45$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

bulunur

b) $\text{Cos}15^\circ = \text{Cos}(45^\circ - 30^\circ) = \text{Cos}45 \,.\, \text{Cos}30 + \text{Sin}45 \,.\, \text{Sin}30$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

c) $\text{tg}15 = \text{tg}(60^\circ - 45^\circ) = \dfrac{\text{tg}60 - \text{tg}45}{1 + \text{tg}60 \,.\, \text{tg}45} = \dfrac{\sqrt{3} - 1}{1 + \sqrt{3} \,.\, 1}$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$ (Paydayı rasyonel yaparsak)

$$= \frac{(\sqrt{3} - 1)(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{(\sqrt{3} - 1)^2}{3 - 1} = \frac{3 - 2\sqrt{3} + 1}{2}$$

$$= \frac{4 - 2\sqrt{3}}{2} = \frac{2\,(2 - \sqrt{3})}{2} = 2 - \sqrt{3}$$

12 $\text{Sin}75^\circ, \text{Cos}75^\circ, \text{tg}75^\circ$ in değerini hesaplayınız.

ÇÖZÜM : $75^\circ = 45^\circ + 30^\circ$ şeklinde yazarsak,

a) $\text{Sin}(75) = \text{Sin}(45 + 30) = \text{Sin}45 \,.\, \text{Cos}30 + \text{Cos}45 \,.\, \text{Sin}30$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

b) $\text{Cos}75 = \text{Cos}(45 + 30) = \text{Cos}45 \,.\, \text{Cos}30 - \text{Sin}45 \,.\, \text{Sin}30$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

c) $\text{tg}75 = \text{tg}(45 + 30) = \dfrac{\text{tg}45 + \text{tg}30}{1 - \text{tg}45 \,.\, \text{tg}30}$

$$= \frac{1+\frac{\sqrt{3}}{3}}{1-1.\frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}}$$

[Paydayı rasyonel yapalım]

$$\text{tg } 75 = \frac{(3+\sqrt{3})^2}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{9+6\sqrt{3}+3}{9-3}$$

$$= \frac{12+6\sqrt{3}}{6} = \frac{6(2+\sqrt{3})}{6} = 2+\sqrt{3}$$

13. tan 3 θ yı tan θ ya bağlı yazınız.

ÇÖZÜM :

$$\tan(3\theta) = \tan(2\theta+\theta) = \frac{\tan 2\theta + \tan\theta}{1-\tan 2\theta . \tan\theta}$$

$$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1-\frac{2\tan\theta}{1-\tan^2\theta}.\tan\theta} = \frac{\frac{2\tan\theta+\tan\theta-\tan^3\theta}{1-\tan^2\theta}}{\frac{1-\tan^2\theta-2\tan^2\theta}{1-\tan^2\theta}}$$

$$\tan(3\theta) = \frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} \text{ olduğunu buluruz.}$$

14. tan x'i $\tan\frac{x}{2}$ türünden yazınız.

ÇÖZÜM : $x = \frac{x}{2} + \frac{x}{2}$ olarak yazılırsa;

$$\tan x = \tan\left(\frac{x}{2}+\frac{x}{2}\right) = \frac{\tan\frac{x}{2}+\tan\frac{x}{2}}{1-\tan\frac{x}{2}.\tan\frac{x}{2}}$$

$$= \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \text{ bulunur.}$$

15. Sin x'i $\frac{x}{2}$ nin trigonometrik oranı şeklinde yazınız.

ÇÖZÜM : I. Yol : $x = \frac{x}{2} + \frac{x}{2}$ yazarsak,

$$\text{Sin}(x) = \text{Sin}\left(\frac{x}{2}+\frac{x}{2}\right)$$

$$= \text{Sin}\frac{x}{2}.\text{Cos}\frac{x}{2} + \text{Cos}\frac{x}{2}.\text{Sin}\frac{x}{2}$$

$$= 2\,\text{Sin}\frac{x}{2}.\text{Cos}\frac{x}{2} \text{ dir.}$$

II. Yol : Sin 2x = 2 Sin x . Cos x olduğunu biliyoruz.

x yerine x/2 yazarsak;

$$\text{Sin } 2\left(\frac{x}{2}\right) = 2\,\text{Sin}\frac{x}{2}.\text{Cos}\frac{x}{2}$$

$$\text{Sin } x = 2\,\text{Sin}\frac{x}{2}.\text{Cos}\frac{x}{2} \text{ bulunur.}$$

16. Aşağıdakileri gerçekleyiniz.

a) $(\cos\alpha + \sin\alpha)^2 = \sin 2\alpha + 1$

b) $\left(\frac{1+\tan\alpha}{1-\tan\alpha}\right)^2 = \frac{1+\sin 2\alpha}{1-\sin 2\alpha}$

c) $\text{Cotg}\,\alpha + \text{Cotg}\,\beta = \dfrac{\text{Sin}\,(\alpha+\beta)}{\text{Sin}\,\alpha \,.\, \text{Sin}\,\beta}$

d) $\text{Sin}\,(\alpha + \dfrac{\pi}{4}) = \dfrac{\sqrt{2}}{2}\,(\text{Sin}\,\alpha + \text{Cos}\,\alpha)$

ÇÖZÜM :

a) $(\text{Cos}\,\alpha + \text{Sin}\,\alpha)^2 = \text{Cos}^2\,\alpha + 2\,\text{Sin}\,\alpha \,.\, \text{Cos}\,\alpha + \text{Sin}^2\,\alpha$

$= \text{Cos}^2\,\alpha + \text{Sin}^2\,\alpha + 2\,\text{Sin}\,\alpha \,.\, \text{Cos}\,\alpha$

$= 1 + \text{Sin}2\,\alpha$

$= \text{Sin}2\,\alpha + 1$

$[\,(a+b)^2 = a^2 + 2ab + b^2$ idi.]

[$\text{Sin}2\,\alpha = 2\,\text{Sin}\,\alpha \,.\, \text{Cos}\,\alpha$ ve

$\text{Sin}^2\,\alpha + \text{Cos}^2\,\alpha = 1$ olduğunu hatırlayınız.]

b) $\left(\dfrac{1+\tan\alpha}{1-\tan\alpha}\right)^2 = \left(\dfrac{1+\dfrac{\text{Sin}\,\alpha}{\text{Cos}\,\alpha}}{1-\dfrac{\text{Sin}\,\alpha}{\text{Cos}\,\alpha}}\right)^2$

$= \left(\dfrac{\dfrac{\text{Cos}\alpha + \text{Sin}\alpha}{\text{Cos}\alpha}}{\dfrac{\text{Cos}\alpha - \text{Sin}\,\alpha}{\text{Cos}\alpha}}\right)^2 = \dfrac{(\text{Cos}\alpha + \text{Sin}\alpha)^2}{(\text{Cos}\alpha - \text{Sin}\alpha)^2}$

$= \dfrac{\text{Cos}^2\,\alpha + 2\,\text{Cos}\,\alpha \,.\, \text{Sin}\,\alpha + \text{Sin}^2\,\alpha}{\text{Cos}^2\,\alpha - 2\,\text{Cos}\,\alpha \,.\, \text{Sin}\,\alpha + \text{Sin}^2\,\alpha}$ (numerator $= 1$, denominator $= 1$)

$= \dfrac{1 + 2\,\text{Cos}\alpha \,.\, \text{Sin}\,\alpha}{1 - 2\,\text{Cos}\alpha \,.\, \text{Sin}\,\alpha} = \dfrac{1 + \text{Sin}\,2\,\alpha}{1 - \text{Sin}\,2\,\alpha}$ bulunur.

c) $\text{Cotg}\,\alpha + \text{Cotg}\,\beta = \dfrac{\text{Cos}\alpha}{\underset{(\text{Sin}\beta)}{\text{Sin}\alpha}} + \dfrac{\text{Cos}\beta}{\underset{(\text{Sin}\beta)}{\text{Sin}\,\beta}}$ [Paydalarını eşitlersek]

$= \dfrac{\text{Cos}\alpha \,.\, \text{Sin}\beta + \text{Cos}\beta \,.\, \text{Sin}\,\alpha}{\text{Sin}\,\alpha \,.\, \text{Sin}\,\beta} = \dfrac{\text{Sin}\beta \,.\, \text{Cos}\alpha + \text{Cos}\beta.\text{Sin}\alpha}{\text{Sin}\,\alpha \,.\, \text{Cos}\,\beta}$

$= \dfrac{\text{Sin}\,(\beta+\alpha)}{\text{Sin}\,\alpha \,.\, \text{Cos}\,\beta}$ dir.

[$\text{Sin}\,(\alpha+\beta) = \text{Sin}\,\alpha \,.\, \text{Cos}\,\beta + \text{Cos}\,\alpha \,.\, \text{Sin}\,\beta$ (6-1) idi.]

d) $\text{Sin}\,(\alpha + \dfrac{\pi}{4}) = \text{Sin}\,\alpha \,.\, \text{Cos}\,\dfrac{\pi}{4} + \text{Cos}\,\alpha \,.\, \text{Sin}\,\dfrac{\pi}{4}$

$= \text{Sin}\,\alpha \cdot \dfrac{\sqrt{2}}{2} + \text{Cos}\,\alpha \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{2}\,(\text{Sin}\alpha + \text{Cos}\alpha)$

17. Aşağıdakilerin en sade yazımını bulunuz.

a) $\text{Sin}2\,\alpha \,.\, \text{Cos}\,\beta + \text{Cos}\,2\,\alpha \,.\, \text{Sin}\,\beta$

b) $\text{Cos}2\,\alpha \,.\, \text{Cos}\,3\,\beta + \text{Sin}\,2\,\alpha \,.\, \text{Sin}\,3\,\beta$

c) $\text{Sin}50^\circ \,.\, \text{Cos}40^\circ + \text{Cos}50^\circ \,.\, \text{Sin}40^\circ$

d) $\text{Cos}^2 3\alpha - \text{Sin}^2\,3\,\alpha$

e) $2\,\text{Cos}^2\,3\,\alpha - 1$

f) $1 - 2\,\text{Sin}^2\,4\,\alpha$

g) $\text{Sin}\,2\,\theta \,.\, \text{Sin}5\theta - \text{Cos}\,5\,\theta \,.\, \text{Cos}\,2\,\theta$

h) $2\,\text{Sin}\,20^\circ \,.\, \text{Cos}\,20^\circ$

i) $1 - 2\,\text{Cos}^2\,4x$

j) $2\,\text{Sin}^2\,7x - 1$

k) $\text{Sin}\,5\,\alpha \,.\, \text{Cos}\,5\,\alpha$

l) $\cos 7\alpha \cdot \sin 7\alpha$

m) $\sin 7\alpha \cdot \cos 5\alpha - \cos 7\alpha \cdot \sin 5\alpha$

n) $\cos 6\alpha \cdot \cos 4\alpha - \sin 6\alpha \cdot \sin 4\alpha$

o) $\cos 3\alpha \cdot \sin \alpha - \sin 3\alpha \cdot \cos \alpha$

ÇÖZÜM :

a) $\sin(2\alpha + \beta)$ (6-1 den)

b) $\cos(2\alpha - 3\beta)$ (6-6 dan)

c) $\sin(40 + 50) = \sin 90 = 1$

d) $\cos 6\alpha$ (6-10 dan) Veya

$[\cos^2 3\alpha - \sin^2 3\alpha = \cos 3\alpha \cdot \cos 3\alpha - \sin 3\alpha \cdot \sin 3\alpha$
$= \cos(3\alpha + 3\alpha) = \cos 6\alpha]$

e) $\cos 6\alpha$ (6-10-c den)

f) $\cos 8\alpha$ (6-10-b den)

g) $-[\cos 5\theta \cdot \cos 2\theta - \sin 2\theta \cdot \sin 5\theta]$
$= -[\cos(5\theta + 2\theta)] = -\cos 7\theta$ (6-2 den)

h) $\sin 40$ (6-9 dan)

i) $-[2\cos^2 4x - 1] = -[\cos 8x] = -\cos 8x$
(6-10-c den)

j) $-[1 - 2\sin^2 7x] = -\cos 14x$ (6-10-b den)

k) $\sin 5\alpha \cdot \cos 5\alpha = \frac{1}{2}[2 \cdot \sin 5\alpha \cdot \cos 5\alpha] = \frac{1}{2}[\sin 10\alpha]$

[2 ile çarpıp, böldük] (6-9 dan)

l) $\cos 7\alpha \cdot \sin 7\alpha = \frac{1}{2}[2 \cdot \sin 7\alpha \cdot \cos 7\alpha]$

$= \frac{1}{2} \cdot \sin 14\alpha$ (6-9 dan)

m) $\sin(7\alpha - 5\alpha) = \sin 2\alpha$ (6-5 den)

n) $\cos(6\alpha + 4\alpha) = \cos 10\alpha$ (6-2 den)

o) $\cos 3\alpha \cdot \sin \alpha - \sin 3\alpha \cdot \cos \alpha$
$= \sin \alpha \cdot \cos 3\alpha - \cos \alpha \cdot \sin 3\alpha = \sin(\alpha - 3\alpha)$
$= \sin(-2\alpha) = -\sin 2\alpha$ (6-1 den)

18. $\sin^2 x$ ve $\cos^2 x$ 'i $\cos 2x$ cinsinden yazınız.

ÇÖZÜM : a) $\cos 2x = 1 - 2\sin^2 x$ idi. Buradan,

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$ bulunur.

b) $\cos 2x = 2\cos^2 x - 1$ idi. Buradan,

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$ bulunur.

19. $\cos\alpha = \sqrt{\frac{1 + \cos 2\alpha}{2}}$ ve $\sin\alpha = \sqrt{\frac{1 + \cos 2\alpha}{2}}$ olduğunu gösteriniz. $(\alpha < 90°)$

ÇÖZÜM :

a) $\cos 2\alpha = 2\cos^2\alpha - 1$ idi. (6-10-c den)

$2\cos^2\alpha = \cos 2\alpha + 1 \Rightarrow \cos^2\alpha = \frac{\cos 2\alpha + 1}{2}$ ve

$\cos\alpha = \sqrt{\frac{1 + \cos 2\alpha}{2}}$ olur.

b) $\cos 2\alpha = 1 - 2\sin^2\alpha$ idi. (6-10-b den)

$2\sin^2\alpha = 1 - \cos 2\alpha \Rightarrow \sin^2\alpha = \frac{1 - \cos 2\alpha}{2}$ ve

$\sin\alpha = \sqrt{\frac{1 - \cos 2\alpha}{2}}$ dir.

20. a) $\text{Sin}(90+\alpha) = \text{Cos}\,\alpha$

b) $\text{Cos}(90+\alpha) = -\text{Sin}\,\alpha$ olduğunu gösteriniz.

ÇÖZÜM :

a) $\text{Sin}(90+\alpha) = \text{Sin}\,90 \,.\, \text{Cos}\,\alpha + \text{Cos}90 \,.\, \text{Sin}\,\alpha$

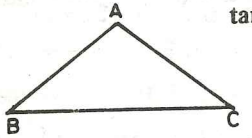
$= 1 \,.\, \text{Cos}\,\alpha + \overbrace{0 \,.\, \text{Sin}\,\alpha}^{0} = \text{Cos}\,\alpha$

b) $\text{Cos}(90+\alpha) = \text{Cos}90 \,.\, \text{Cos}\,\alpha - \text{Sin}90 \,.\, \text{Sin}\,\alpha$

$= \overbrace{0 \,.\, \text{Cos}\,\alpha}^{0} - 1 \,.\, \text{Sin}\,\alpha = -\text{Sin}\,\alpha$ çıkar.

21. $\widehat{ABC}$ de $tgA + tgB + tgC = tgA \,.\, tgB \,.\, tgC$ olduğunu gösteriniz.

ÇÖZÜM : $\hat{A} + \hat{B} = 180 - \hat{C}$ dersek, eşitliğin her iki tarafının tanjantını alabiliriz. (Sayılar eşit ise tanjantlarında eşittir.)



$\widehat{ABC}$ de $m\hat{A} + m\hat{B} + m\hat{C} = 180°$ idi.

($m\hat{A} \equiv A$ olarak yazacağız.)

$tg(A+B) = tg(180-C)$

$$\frac{tgA + tgB}{1 - tgA \,.\, tgB} = -\,tg\,C \qquad [\, tg(180-\alpha) = -\,tg\,\alpha \text{ idi} \,]$$

$tg\,A + tg\,B = -\,tg\,C\,(1 - tgA \,.\, tgB)$

$tg\,A + tg\,B = -\,tg\,C + tg\,A \,.\, tgB \,.\, tg\,C$

$tg\,A + tg\,B + tg\,C = tg\,A \,.\, tgB \,.\, tgC$ bulunur.

22. $tg\,A = 2$ ise, $tg2A$ ve $tg4A$ yı hesaplayınız.

a) $$tg2A = \frac{2\,tg\,A}{1 - tg^2 A} = \frac{2\,.\,(2)}{1-(2)^2} = \frac{4}{1-4} = -\frac{4}{3}$$

b) $$tg4A = tg\,(2\,.\,(2A)) = \frac{2\,tg\,2A}{1 - tg^2 2A} = \frac{2 - \frac{4}{3}}{1 - (-\frac{4}{3})^2}$$

$$= \frac{-\frac{8}{3}}{1 - \frac{16}{9}} = \frac{24}{7} \text{ dir.}$$

23. Aşağıdaki ifadeleri sadeleştiriniz.

a) $\text{Cos}(\alpha+\beta) - \text{Cos}(\alpha-\beta)$

b) $\text{Sin}(\alpha+\beta) - \text{Sin}(\alpha-\beta)$

c) $\text{Sin}(\alpha+\beta) \,.\, \text{Cos}(\alpha+\theta) - \text{Cos}(\alpha+\beta) \,.\, \text{Sin}(\alpha+\theta)$

d) $(\text{Sin}\,\frac{\pi}{12} - \text{Cos}\,\frac{\pi}{12})^2$

e) $\text{Cos}^2(\frac{\pi}{4} - \alpha) - \text{Sin}^2(\frac{\pi}{4} - \alpha)$

ÇÖZÜM : a) $\text{Cos}(\alpha+\beta) - \text{Cos}(\alpha-\beta)$

$= [\text{Cos}\alpha \,.\, \text{Cos}\beta - \text{Sin}\alpha \,.\, \text{Sin}\beta\,] - [\,\text{Cos}\alpha.\text{Cos}\beta + \text{Sin}\alpha.\text{Sin}\beta]$

$= -2\,\text{Sin}\,\alpha \,.\, \text{Sin}\,\beta$

UYARI : $\text{Cos}(\alpha+\beta) - \text{Cos}(\alpha-\beta) = -2\,\text{Sin}\,\alpha \,.\, \text{Sin}\,\beta$ ise,

$\text{Sin}\,\alpha \,.\, \text{Sin}\,\beta = +\frac{1}{2}[\,\text{Cos}(\alpha-\beta) - \text{Cos}(\alpha+\beta)\,]$

yazılabilir mi ?

b) $\text{Sin}(\alpha+\beta) - \text{Sin}(\alpha-\beta)$

$= [\text{Sin}\alpha \,.\, \text{Cos}\beta + \text{Cos}\alpha \,.\, \text{Sin}\beta] - [\text{Sin}\alpha \,.\, \text{Cos}\beta - \text{Cos}\alpha\,\text{Sin}\beta]$

$= 2\,\text{Cos}\,\alpha \,.\, \text{Sin}\,\beta$

UYARI :

$\text{Cos}\,\alpha \,.\, \text{Sin}\,\beta = \frac{1}{2}\,[\,\text{Sin}(\alpha+\beta) - \text{Sin}(\alpha-\beta)\,]$

yazabilirmiyiz ?

c) $\text{Sin}\underbrace{(\alpha+\beta)}_{p} \,.\, \text{Cos}\underbrace{(\alpha+\theta)}_{q} - \text{Cos}\underbrace{(\alpha+\beta)}_{p} \,.\, \text{Sin}\underbrace{(\alpha+\theta)}_{q}$

$= \text{Sin}\,p \,.\, \text{Cos}\,q - \text{Cos}\,p \,.\, \text{Sin}\,q = \text{Sin}(p-q)$

$= \text{Sin}\,[\,(\alpha+\beta) - (\alpha+\theta)\,] = \text{Sin}(\beta-\theta)$

d) $(\text{Sin}\frac{\pi}{12} - \text{Cos}\frac{\pi}{12})^2$

$= \text{Sin}^2\frac{\pi}{12} - 2\,.\,\text{Sin}\frac{\pi}{12}\,.\,\text{Cos}\frac{\pi}{12} + \text{Cos}^2\frac{\pi}{12}$

$= \overbrace{\text{Sin}^2\frac{\pi}{12} + \text{Cos}^2\frac{\pi}{12}}^{1} - 2\,\text{Sin}\frac{\pi}{12}\,\text{Cos}\frac{\pi}{12}$

$= 1 - \text{Sin}(2\circ\frac{\pi}{12}) = 1 - \text{Sin}(\frac{\pi}{6}) = 1 - \text{Sin}\,30^\circ$

$= 1 - \frac{1}{2} = \frac{1}{2}$ [$\text{Sin}2\alpha = 2\text{Sin}\,\alpha \,.\, \text{Cos}\alpha$ idi.

$\text{Sin}\frac{\pi}{6} = 2\,.\,\text{Sin}\frac{\pi}{12}\,.\,\text{Cos}\frac{\pi}{12}$

olduğunu görünüz.]

e) $\text{Cos}^2\underbrace{(\frac{\pi}{4}-\alpha)}_{p} - \text{Sin}^2\underbrace{(\frac{\pi}{4}-\alpha)}_{p}$

$= \text{Cos}^2 p - \text{Sin}^2 p = \text{Cos}2p$ [$\pi/4 - \alpha = p$ diyelim.]

$= \text{Cos}\,[\,2\,(\frac{\pi}{4}-\alpha)] = \text{Cos}\,[\frac{\pi}{2} - 2\alpha]$

$= \text{Cos}\,(90 - 2\alpha) = \text{Sin}2\alpha$ [$\text{Cos}(90-\alpha) = \text{Sin}\,\alpha$ idi]

24. Aşağıdaki özdeşlikleri gerçekleyiniz.

a) $\text{Cos}(\alpha+\beta) + (\text{Cos}\,\alpha - \beta) = 2\,\text{Cos}\,\alpha \,.\, \text{Cos}\,\beta$

b) $\text{Sin}(\alpha+\beta) + \text{Sin}(\alpha-\beta) = 2\,\text{Sin}\,\alpha \,.\, \text{Cos}\,\beta$

ÇÖZÜM :

a) $\text{Cos}(\alpha+\beta) = \text{Cos}\,\alpha \,.\, \text{Cos}\,\beta - \text{Sin}\,\alpha \,.\, \text{Sin}\,\beta$

$+\ \text{Cos}(\alpha-\beta) = \text{Cos}\,\alpha \,.\, \text{Cos}\,\beta + \text{Sin}\,\alpha \,.\, \text{Sin}\,\beta$

$\text{Cos}(\alpha-\beta) + \text{Cos}(\alpha-\beta) = 2\,\text{Cos}\alpha \,.\, \text{Cos}\beta$

} Taraf tarafa toplayalım;

NOT : $\text{Cos}\,\alpha \,.\, \text{Cos}\,\beta = \frac{1}{2}\,[\,\text{Cos}(\alpha+\beta) + \text{Cos}(\alpha-\beta)\,]$

yazabilirmiyiz ?

b) $\text{Sin}(\alpha+\beta) = \text{Sin}\,\alpha \,.\, \text{Cos}\,\beta + \text{Cos}\,\alpha \,.\, \text{Sin}\,\beta$

$\pm\ \text{Sin}(\alpha-\beta) = \text{Sin}\,\alpha \,.\, \text{Cos}\,\beta - \text{Cos}\,\alpha \,.\, \text{Sin}\,\beta$

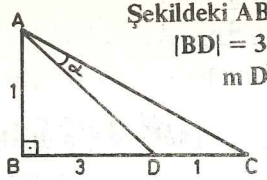
$\text{Sin}(\alpha+\beta) + \text{Sin}(\alpha-\beta) = 2\,\text{Sin}\alpha \,.\, \text{Cos}\,\beta$

} Taraf tarafa toplayalım ;

NOT : $\text{Sin}\,\alpha \,.\, \text{Cos}\,\beta = \frac{1}{2}\,[\,\text{Sin}(\alpha+\beta) + \text{Sin}(\alpha-\beta)\,]$

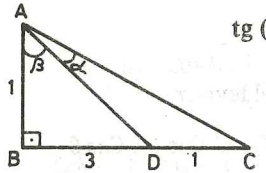
yazabilirmiyiz ?

25. Şekildeki ABC dik üçgeninde $|AB| = 1$, $|BD| = 3$, $|DC| = 1$ ve $m\hat{B} = 90°$ dir. $m\,\hat{DAC} = \alpha$ ise, $tg\,\alpha$ nedir ?



ÇÖZÜM :

$m\,\hat{BAD} = \beta$ dersek, $m\,\hat{BAC} = \alpha + \beta$ olur.



$$tg(\alpha+\beta) = \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha \,.\, tg\,\beta}$$

olur. (6 - 3 den)

Şekilde $tg(\alpha+\beta) = \frac{4}{1} = 4$

$tg\,\beta = \frac{3}{1} = 3$ olduğunu görünüz.

$$\frac{4}{1} = \frac{tg\,\alpha + 3}{1 - 3\,tg\,\alpha} \Rightarrow$$

$4(1 - 3\,tg\,\alpha) = tg\,\alpha + 3$; $4 - 12\,tg\,\alpha = tg\,\alpha + 3$,

$1 = 13\,tg\,\alpha$, $tg\,\alpha = \frac{1}{13}$ bulunur.

26. $\sin x - \cos x = \frac{1}{3}$ ise, $\sin 2x$ in değerini bulunuz.

ÇÖZÜM : Her iki tarafın karesini alırsak;

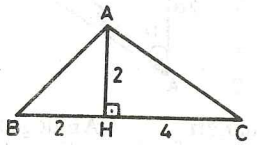
$(\sin x - \cos x)^2 = (\frac{1}{3})^2 \Rightarrow \sin^2 x - 2 \sin x \,.\, \cos x + \cos^2 x$

$$= \frac{1}{9} \Rightarrow \overbrace{\sin^2 x + \cos^2 x}^{1} - 2 \sin x \,.\, \cos x = \frac{1}{9}$$

$1 - 2 \sin x \,.\, \cos x = \frac{1}{9}$, $2 \sin x \,.\, \cos x = 1 - \frac{1}{9}$

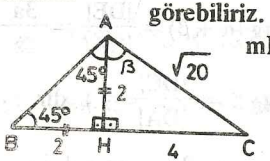
$\boxed{\sin 2x = \frac{8}{9}}$, [$\sin 2x = 2 \sin x \,.\, \cos x$ idi]

27.



Şekildeki ABC üçgeninde; $|AH| = |BH| = 2$ ve $|HC| = 4$ dür. $m\,\hat{BAC} = \alpha$ ise, $\sin\alpha$ nın değeri nedir ?

ÇÖZÜM : ABH üçgeninin ikizkenar dik üçgen olduğunu görebiliriz. O halde,



$m\hat{BAH} = m\hat{HBA} = 45°$ dir.

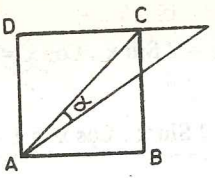
$m\,\hat{HAC} = \beta$ dersek,

$\alpha = 45° + \beta$ olur.

$\sin\alpha = \sin(45 + \beta) = \sin 45 \,.\, \cos\beta + \cos 45 \,.\, \sin\beta$ (6 - 1 den)

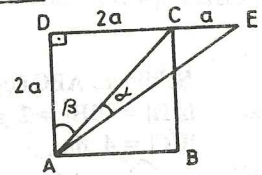
$$= \frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{20}} + \frac{\sqrt{2}}{2} \cdot \frac{4}{\sqrt{20}}$$

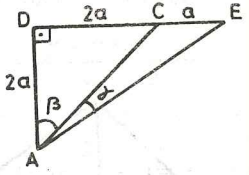
$$= \frac{\sqrt{2}}{\sqrt{20}} + \frac{\sqrt{2}\,.\,2}{\sqrt{20}} = \frac{3\sqrt{2}}{\sqrt{20}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

28. 

Şekilde ABCD kare ve [DE] // [AB] dir. | DC | = 2 . | CE | ise, tg α nedir ?

ÇÖZÜM :





|EC| = a dersek karenin kenarları 2a olur. $m\widehat{DAC} = \beta$ diyelim.

DAE üçgeni bize soruyu çözmek için yeterlidir.

$$tg(\alpha+\beta) = \frac{|DE|}{|DA|} = \frac{3a}{2a} = \frac{3}{2}$$

$$tg\,\beta = \frac{|DC|}{|DA|} = 1 \text{ dir.}$$

$$tg(\alpha+\beta) = \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha \,.\, tg\,\beta} \Rightarrow \frac{3}{2} = \frac{tg\,\alpha + 1}{1 - tg\,\alpha} \Rightarrow$$

$$\Rightarrow 3(1 - tg\,\alpha) = 2(tg\,\alpha + 1) \Rightarrow$$

$$3 - 3\,tg\,\alpha = 2\,tg\,\alpha + 2\,, \quad 1 = 5\,tg\,\alpha\,, \quad tg\,\alpha = \frac{1}{5} \text{ dir.}$$

7. BÖLÜM

DÖNÜŞÜM ve TERS DÖNÜŞÜM FORMÜLLERİ

A) ÇARPIMI, TOPLAMA DÖNÜŞTÜRME

(7-1) $\sin\alpha \,.\, \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

(7-2) $\cos\alpha \,.\, \sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

(7-3) $\cos\alpha \,.\, \cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

(7-4) $\sin\alpha \,.\, \sin\beta = \frac{-1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$

B) TOPLAMI, ÇARPIMA DÖNÜŞTÜRME

(7-5) $\sin\alpha + \sin\beta = 2\sin(\frac{\alpha+\beta}{2}) \,.\, \cos(\frac{\alpha-\beta}{2})$

(7-6) $\sin\alpha - \sin\beta = 2\sin(\frac{\alpha-\beta}{2}) \,.\, \cos(\frac{\alpha+\beta}{2})$

(7-7) $\cos\alpha + \cos\beta = 2\cos(\frac{\alpha+\beta}{2}) \,.\, \cos(\frac{\alpha-\beta}{2})$

(7-8) $\cos\alpha - \cos\beta = -2\sin(\frac{\alpha+\beta}{2}) \,.\, \sin(\frac{\alpha-\beta}{2})$

NOT: Problemlerde en sık kullanılan özellikler, (7-5), (7-6), (7-7), (7-8) ve (7-3) dür.

ALIŞTIRMALAR

1. a) $\sin\alpha \cdot \cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$

b) $\cos\alpha \cdot \sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$

c) $\cos\alpha \cdot \cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$

d) $\sin\alpha \cdot \sin\beta = -\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)]$

olduğunu gösteriniz.

ÇÖZÜM :

a) 6. bölüm 24. sorunun b şıkkında gösterildi.
b) 6. bölüm 23. sorunun b şıkkında gösterildi.
c) 6. bölüm 24. sorunun a şıkkında gösterildi.
d) 6. bölüm 23. sorunun a şıkkında gösterildi.

2. Aşağıdakileri gerçekleyiniz.

a) $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha+\beta}{2}$

b) $\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2} \cdot \cos\frac{\alpha+\beta}{2}$

c) $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$

d) $\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}$

ÇÖZÜM :

a) $\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$
$\sin(\alpha-\beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$ idi. Taraf tarafa toplarsak;

(*) $\sin(\alpha+\beta)+\sin(\alpha-\beta) = 2\sin\alpha \cdot \cos\beta$ olur.

$\alpha+\beta = x$
$\alpha-\beta = y$ diyelim.

$$\begin{array}{r} \alpha+\beta=x \\ +\ \alpha-\beta=y \\ \hline 2\alpha = x+y \end{array}$$

$$\boxed{\alpha = \frac{x+y}{2}} \text{ ve } \boxed{\beta = \frac{x-y}{2}}$$

olur. Bulduklarımızı (*) da yerine koyalım;

$$\sin\left(\frac{x+y}{2}+\frac{x-y}{2}\right)+\sin\left(\frac{x+y}{2}-\frac{x-y}{2}\right)$$

$$= 2\sin\frac{x+y}{2} \cdot \cos\frac{x-y}{2}$$

$$\boxed{\sin x + \sin y = 2\sin\frac{x+y}{2} \cdot \cos\frac{x-y}{2}} \text{ bulunur.}$$

b) $\left.\begin{array}{l}\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta \\ \sin(\alpha-\beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta\end{array}\right\}$ idi. Taraf tarafa çıkarırsak;

(**) $\sin(\alpha+\beta)-\sin(\alpha-\beta) = 2\cos\alpha \cdot \sin\beta$ olur.

$\alpha+\beta = x$ dersek
$\alpha-\beta = y$

$$\alpha = \frac{x+y}{2}$$

$$\beta = \frac{x-y}{2} \text{ olur.}$$

Bulduklarımızı (**) de yerine koyalım.

$$\text{Sin}\left(\frac{x+y}{2}+\frac{x-y}{2}\right)-\text{Sin}\left(\frac{x+y}{2}-\frac{x-y}{2}\right)$$

$$=2\,\text{Cos}\left(\frac{x+y}{2}\right).\,\text{Sin}\left(\frac{x-y}{2}\right)$$

$$\boxed{\text{Sin } x-\text{Sin } y=2\,\text{Cos}\,\frac{x+y}{2}.\,\text{Sin}\,\frac{x-y}{2}}\quad\text{olur.}$$

c) $\left.\begin{array}{l}\text{Cos}(\alpha+\beta)=\text{Cos}\,\alpha.\,\text{Cos}\,\beta-\text{Sin}\,\alpha.\,\text{Sin}\,\beta\\ \pm\ \text{Cos}(\alpha-\beta)=\pm\ \text{Cos}\,\alpha.\,\text{Cos}\,\beta+\text{Sin}\,\alpha.\,\text{Sin}\,\beta\end{array}\right\}$ idi. Taraf tarafa toplarsak;

(***) $\text{Cos}(\alpha+\beta)+\text{Cos}(\alpha-\beta)=2\,\text{Cos}\,\alpha.\,\text{Cos}\,\beta$ olur.

$\alpha+\beta=x$
$\alpha-\beta=y$ dersek $\quad\alpha=\dfrac{x+y}{2}$

$$\beta=\frac{x-y}{2}\quad\text{olur.}$$

Bunları (***) de yerine yazarsak,

$$\boxed{\text{Cos}(x)+\text{Cos}(y)=2\,\text{Cos}\left(\frac{x+y}{2}\right).\,\text{Cos}\left(\frac{x-y}{2}\right)}$$

d) $\left.\begin{array}{l}\text{Cos}(\alpha+\beta)=\text{Cos}\,\alpha.\,\text{Cos}\,\beta-\text{Sin}\,\alpha.\,\text{Sin}\,\beta\\ -\ \text{Cos}(\alpha-\beta)=-\ \text{Cos}\,\alpha.\,\text{Cos}\,\beta+\text{Sin}\,\alpha.\,\text{Sin}\,\beta\end{array}\right\}$ idi. Taraf tarafa çıkarırsak;

(****) $\text{Cos}(\alpha+\beta)-\text{Cos}(\alpha-\beta)=-2\,\text{Sin}\,\alpha.\,\text{Sin}\,\beta$ olur.

$\alpha+\beta=x$
$\alpha-\beta=y$ için $\quad\alpha=\dfrac{x+y}{2}$

$$\beta=\frac{x-y}{2}\quad\text{olur.}$$

Değerlerini (****) de yerine yazarsak,

$$\boxed{\text{Cos } x-\text{Cos } y=-2\,\text{Sin}\,\frac{x+y}{2}.\,\text{Sin}\,\frac{x-y}{2}}$$

3. $\text{tg}\,\alpha+\text{tg}\,\beta=\dfrac{\text{Sin}(\alpha+\beta)}{\text{Cos}\,\alpha.\,\text{Cos}\,\beta}$ olduğunu gösteriniz.

ÇÖZÜM :

$$\text{tg}\,\alpha+\text{tg}\,\beta=\frac{\text{Sin}\,\alpha}{\text{Cos}\alpha}+\frac{\text{Sin}\,\beta}{\text{Cos}\,\beta}=\frac{\text{Sin}\alpha.\,\text{Cos}\beta+\text{Cos}\alpha.\text{Sin}\beta}{\text{Cos}\,\alpha.\,\text{Cos}\,\beta}$$

$$=\frac{\text{Sin}(\alpha+\beta)}{\text{Cos}\,\alpha.\,\text{Cos}\,\beta}\quad\text{dır.}$$

4. $\text{tg}\,\alpha-\text{tg}\,\beta=\dfrac{\text{Sin}(\alpha-\beta)}{\text{Cos}\,\alpha.\,\text{Cos}\,\beta}$ olduğunu gösteriniz.

ÇÖZÜM :

$$\text{tg}\,\alpha-\text{tg}\,\beta=\frac{\text{Sin}\,\alpha}{\text{Cos}\,\alpha}-\frac{\text{Sin}\,\beta}{\text{Cos}\,\beta}=\frac{\text{Sin}\alpha.\,\text{Cos}\beta-\text{Cos}\alpha.\text{Sin}\beta}{\text{Cos}\,\alpha.\,\text{Cos}\,\beta}$$

$$=\frac{\text{Sin}(\alpha-\beta)}{\text{Cos}\,\alpha.\,\text{Cos}\,\beta}\quad\text{olur.}$$

5. $\text{Cotg}\,\alpha + \text{Cotg}\,\beta = \dfrac{\text{Sin}\,(\alpha+\beta)}{\text{Sin}\,\alpha\,.\,\text{Sin}\,\beta}$ olduğunu gösteriniz.

ÇÖZÜM : $\text{Cotg}\,\alpha + \text{Cotg}\,\beta = \dfrac{\text{Cos}\,\alpha}{\text{Sin}\,\alpha} + \dfrac{\text{Cos}\,\beta}{\text{Sin}\,\beta}$

$$= \frac{\text{Sin}\beta\,.\,\text{Cos}\alpha + \text{Cos}\beta\,.\,\text{Sin}\alpha}{\text{Sin}\,\alpha\,.\,\text{Sin}\,\beta} = \frac{\text{Sin}(\alpha+\beta)}{\text{Sin}\,\alpha\,.\,\text{Sin}\,\beta}$$

6. $\text{tg}\,\alpha + \text{Cotg}\,\beta = \dfrac{\text{Cos}\,(\alpha-\beta)}{\text{Cos}\,\alpha\,.\,\text{Sin}\,\beta}$ olduğunu gösteriniz.

ÇÖZÜM :

$$\text{tg}\,\alpha + \text{tg}\,\beta = \frac{\text{Sin}\,\alpha}{\text{Cos}\,\alpha} + \frac{\text{Cos}\,\beta}{\text{Sin}\,\beta} = \frac{\text{Sin}\alpha\,.\,\text{Sin}\beta + \text{Cos}\alpha.\text{Cos}\beta}{\text{Cos}\,\alpha\,.\,\text{Sin}\,\beta}$$

$$= \frac{\text{Cos}\alpha\,.\,\text{Cos}\beta + \text{Sin}\alpha\,.\,\text{Sin}\beta}{\text{Cos}\,\alpha\,.\,\text{Sin}\,\beta} = \frac{\text{Cos}\,(\alpha-\beta)}{\text{Cos}\,\alpha\,\cdot\,\text{Sin}\,\beta}$$

7. $\dfrac{\text{Sin}\,\alpha + \text{Sin}\,\beta}{\text{Sin}\,\alpha - \text{Sin}\,\beta} = \dfrac{\text{tg}\,(\frac{\alpha+\beta}{2})}{\text{tg}\,(\frac{\alpha-\beta}{2})}$ olduğunu gösteriniz.

ÇÖZÜM :

$$\text{Sin}\alpha + \text{Sin}\beta = 2\,\text{Sin}\,\frac{\alpha+\beta}{2}\,.\,\text{Cos}\,\frac{\alpha-\beta}{2} \quad \text{(7-5 den)}$$

$$\text{Sin}\alpha - \text{Sin}\beta = 2\,\text{Sin}\,\frac{\alpha-\beta}{2}\,.\,\text{Cos}\,\frac{\alpha+\beta}{2} \quad \text{(7-6 dan)}$$

$$\frac{\text{Sin}\alpha + \text{Sin}\beta}{\text{Sin}\alpha - \text{Sin}\beta} = \frac{\text{Sin}\,\frac{\alpha+\beta}{2}\,.\,\text{Cos}\,\frac{\alpha-\beta}{2}}{\text{Cos}\,\frac{\alpha+\beta}{2}\,.\,\text{Sin}\,\frac{\alpha-\beta}{2}}$$

$$= \frac{\text{Sin}\,\frac{\alpha+\beta}{2}}{\text{Cos}\,\frac{\alpha+\beta}{2}} \cdot \frac{1}{\frac{\text{Sin}\,\frac{\alpha-\beta}{2}}{\text{Cos}\,\frac{\alpha-\beta}{2}}}$$

$$= \text{tg}\,\frac{\alpha+\beta}{2} \cdot \frac{1}{\text{tg}\,\frac{\alpha-\beta}{2}} = \frac{\text{tg}\,(\frac{\alpha+\beta}{2})}{\text{tg}\,(\frac{\alpha-\beta}{2})}$$

8. Aşağıdakileri çarpım şeklinde yazınız.

a) $\text{Sin}\,4\,\alpha + \text{Sin}\,2\,\alpha$ b) $\text{Cos}\,4\,\alpha + \text{Cos}\,2\,\alpha$

c) $\text{Sin}\,8\,\alpha - \text{Sin}\,6\,\alpha$ d) $\text{Cos}\,8\,\alpha - \text{Cos}\,6\,\alpha$

e) $\text{Cos}\,70 + \text{Cos}\,20$ f) $\text{Cos}\,70 - \text{Cos}\,20$

g) $\text{Sin}\,55 + \text{Sin}\,25$ h) $\text{Cos}\,25 - \text{Cos}\,65$

ÇÖZÜM :

a) $\text{Sin}\,4\,\alpha + \text{Sin}\,2\,\alpha = 2\,\text{Sin}\,\frac{4\alpha+2\alpha}{2}\,.\,\text{Cos}\,\frac{4\alpha-2\alpha}{2}$

$= 2\,\text{Sin}\,3\,\alpha\,.\,\text{Cos}\,\alpha$

b) $\text{Cos}\,4\,\alpha + \text{Cos}\,2\,\alpha = 2\,\text{Cos}\,\frac{4\alpha+2\alpha}{2}\,.\,\text{Cos}\,\frac{4\alpha-2\alpha}{2}$

$= 2\,\text{Cos}\,3\,\alpha\,.\,\text{Cos}\,\alpha$

c) $\text{Sin}\,8\,\alpha - \text{Sin}\,6\,\alpha = 2\,\text{Sin}\,\frac{8\alpha-6\alpha}{2}\,.\,\text{Cos}\,\frac{8\alpha+6\alpha}{2}$

$= 2\,\text{Sin}\,\alpha\,.\,\text{Cos}\,7\,\alpha$

d) $\text{Cos}8\alpha - \text{Cos}6\alpha = -\,2\,\text{Sin}\,\frac{8\alpha+6\alpha}{2}\,.\,\text{Sin}\,\frac{8\alpha-6\alpha}{2}$

$= -\,2\,\text{Sin}\,7\,\alpha\,.\,\text{Sin}\,\alpha$

e) $\text{Cos}70 + \text{Cos}20 = 2\,\text{Cos}\dfrac{70+20}{2}.\text{Cos}\dfrac{70-20}{2}$

$= 2\,\text{Cos}\,45 . \text{Cos}\,25 = \sqrt{2}\,\text{Cos}\,25$

f) $\text{Cos}70 - \text{Cos}20 = -2\,\text{Sin}\dfrac{70+20}{2}.\text{Sin}\dfrac{70-20}{2}$

$= -2\,\text{Sin}\,45 . \text{Sin}\,25 = -\sqrt{2}\,\text{Sin}\,25$

g) $\text{Sin}55 + \text{Sin}25 = 2\,\text{Sin}\dfrac{55+25}{2}.\text{Cos}\dfrac{55-25}{2}$

$= 2\,\text{Sin}\,40 . \text{Cos}\,15$

h) $\text{Cos}25 - \text{Cos}65 = -2\,\text{Sin}\dfrac{25+65}{2}.\text{Sin}\dfrac{25-65}{2}$

$= -2\,\text{Sin}45 . \text{Sin}\,(-20) = \sqrt{2}.\text{Sin}20$

9. Aşağıdakileri en sade şekilde yazınız.

a) $\dfrac{\text{Sin}5x + \text{Sin}x}{\text{Cos}5x + \text{Cos}x}$ b) $\dfrac{\text{Sin}7x - \text{Sin}3x}{\text{Cos}7x + \text{Cos}3x}$

c) $\dfrac{\text{Cos}6x - \text{Cos}2x}{\text{Sin}6x - \text{Sin}2x}$ d) $\dfrac{\text{Cos}x - \text{Cos}10x}{\text{Sin}x - \text{Sin}\,10\,x}$

ÇÖZÜM :

a) $\dfrac{\text{Sin}5x + \text{Sin}x}{\text{Cos}5x + \text{Cos}x} = \dfrac{2\,\text{Sin}\dfrac{5x+x}{2}.\text{Cos}\dfrac{5x-x}{2}}{2\,\text{Cos}\dfrac{5x+x}{2}.\text{Cos}\dfrac{5x-x}{2}}$

$= \dfrac{\text{Sin}3x . \cancel{\text{Cos}2x}}{\text{Cos}3x . \cancel{\text{Cos}2x}} = \text{tg}3x$

b) $\dfrac{\text{Sin}7x - \text{Sin}3x}{\text{Cos}7x + \text{Cos}3x} = \dfrac{2\,\text{Sin}\dfrac{7x-3x}{2}.\text{Cos}\dfrac{7x+3x}{2}}{2\,\text{Cos}\dfrac{7x+3x}{2}.\text{Cos}\dfrac{7x-3x}{2}}$

$= \dfrac{\text{Sin}2x . \cancel{\text{Cos}5x}}{\cancel{\text{Cos}5x} . \text{Cos}2x} = \text{tg}2x$

c) $\dfrac{\text{Cos}6x - \text{Cos}2x}{\text{Sin}6x - \text{Sin}2x} = \dfrac{-2\,\text{Sin}\dfrac{6x+2x}{2}.\text{Sin}\dfrac{6x-2x}{2}}{2\,\text{Sin}\dfrac{6x-2x}{2}.\text{Cos}\dfrac{6x+2x}{2}}$

$= \dfrac{-\text{Sin}4x . \cancel{\text{Sin}2x}}{\cancel{\text{Sin}2x} . \text{Cos}4x} = -\text{tg}4x$

d) $\dfrac{\text{Cos}x - \text{Cos}10x}{\text{Sin}x - \text{Sin}\,10\,x} = \dfrac{-2\,\text{Sin}\dfrac{x+10x}{2}.\text{Sin}\dfrac{x-10x}{2}}{2\,\text{Sin}\dfrac{x-10x}{2}.\text{Cos}\dfrac{x+10x}{2}}$

$= \dfrac{-\text{Sin}\left(\dfrac{11x}{2}\right).\text{Sin}\left(-\dfrac{9x}{2}\right)}{\text{Sin}\left(-\dfrac{9x}{2}\right).\text{Cos}\left(\dfrac{11x}{2}\right)} = -\text{tg}\left(\dfrac{11x}{2}\right)$

10. Aşağıdakileri çarpım şeklinde yazınız.

a) $\text{Sin}2x + \text{Sin}4x + \text{Sin}6x$ b) $1 + \text{Cos}2x + \text{Cos}4x$

c) $\text{Sin}7x - \text{Sin}3x - \text{Sin}x$ d) $\text{Sin}9x + \text{Sin}5x - \text{Sin}7x$

ÇÖZÜM :

a) $(\text{Sin}6x + \text{Sin}2x) + \text{Sin}4x$

$= 2 \text{ Sin } \frac{6x + 2x}{2} . \text{ Cos } \frac{6x - 2x}{2} + \text{Sin}4x$

$= 2 \text{ Sin } 4x . \text{Cos}2x + \text{Sin}4x$

$= \text{Sin } 4x \, [\, 2 \text{ Cos } 2x + 1 \,]$

b) $(\text{Cos}4x + \text{Cos}0) + \text{Cos}2x$

$= 2 \text{ Cos } \frac{4x + 0}{2} . \text{ Cos } \frac{4x - 0}{2} + \text{Cos}2x$

$= 2 \text{ Cos } 2x . \text{ Cos } 2x + \text{Cos } 2x$

$= \text{Cos}2x \, (2 \text{ Cos } 2x + 1)$

[$1 = \text{Cos } 0$ aldık.]

c) $(\text{Sin}7x - \text{Sin}x) - \text{Sin } 3x$

$= 2 \text{ Sin } \frac{7x - x}{2} . \text{ Cos } \frac{7x + x}{2} - \text{Sin } 3x$

$= 2 \text{ Sin } 3x . \text{ Cos } 4x - \text{Sin } 3x$

$= \text{Sin } 3x \, (2 \text{ Cos } 4x - 1)$

d) $(\text{Sin}9x + \text{Sin}5x) - \text{Sin}7x = 2 \text{ Sin}7x . \text{Cos}2x - \text{Sin}7x$

$= \text{Sin}7x \, (2 \text{ Cos}2x - 1)$

11. Aşağıdaki ifadeleri sadeleştiriniz.

a) $\frac{\text{Sin}\alpha + \text{Sin}\beta}{\text{Sin } (\alpha + \beta)}$

b) $\frac{\text{Sin } 5\,\alpha - \text{Sin } 3\,\alpha}{\text{Cos } 5\,\alpha + \text{Cos}3\,\alpha}$

c) $\frac{\text{Sin}2\,\alpha + \text{Sin}4\alpha + \text{Sin}6\alpha}{1 + \text{Cos } 2\,\alpha + \text{Cos}4\,\alpha}$

d) $\frac{\text{Cos } (\alpha + \beta) - \text{Cos}(\alpha - \beta)}{\text{Sin } (\alpha + \beta) + \text{Sin}(\alpha - \beta)}$

ÇÖZÜM :

a) $\text{Sin}2\alpha = 2 \text{ Sin } \alpha . \text{ Cos } \alpha$ idi. Buradan

$\text{Sin } \alpha = 2 \text{ Sin } \frac{\alpha}{2} . \text{ Cos } \frac{\alpha}{2}$ yazabiliyorduk. O halde

$\text{Sin } (\alpha + \beta) = 2 \text{ Sin } (\frac{\alpha + \beta}{2}) . \text{ Cos } (\frac{\alpha + \beta}{2})$

yazabiliriz. Buradan;

$$\frac{\text{Sin } \alpha + \text{Sin } \beta}{\text{Sin } (\alpha + \beta)} = \frac{2 \text{ Sin } (\frac{\alpha + \beta}{2}) . \text{ Cos } (\frac{\alpha - \beta}{2})}{2 \text{ Sin } (\frac{\alpha + \beta}{2}) . \text{ Cos } (\frac{\alpha + \beta}{2})}$$

$$= \frac{\text{Cos } (\frac{\alpha - \beta}{2})}{\text{Cos } (\frac{\alpha + \beta}{2})} \quad \text{bulunur.}$$

DİKKAT : Bazı öğrenciler $\text{Sin } \alpha + \text{Sin } \beta = \text{Sin } (\alpha + \beta)$ *yazarak cevabı 1 bulurlar.*

$$\text{Sin } \alpha + \text{Sin } \beta \neq \text{Sin } (\alpha + \beta)$$

dikkat ediniz.

b) $$\frac{\text{Sin}5\,\alpha - \text{Sin } 3\,\alpha}{\text{Cos}5\,\alpha + \text{Cos}3\,\alpha} = \frac{2 \text{ Sin } \frac{5\alpha - 3\alpha}{2} . \text{ Cos } \frac{5\alpha + 3\alpha}{2}}{2 \text{ Cos } \frac{5\alpha + 3\alpha}{2} . \text{ Cos } \frac{5\alpha - 3\alpha}{2}}$$

$$= \frac{\text{Sin } \alpha \, . \, \text{Cos } 4\,\alpha}{\text{Cos } 4\,\alpha \, . \, \text{Cos } \alpha} = \text{tg } \alpha$$

c) $\dfrac{(\text{Sin}6\alpha+\text{Sin}2\alpha)+\text{Sin}4\alpha}{(\text{Cos}4\alpha+\text{Cos}0)+\text{Cos}\,2\alpha}=\dfrac{2\text{Sin}4\alpha\,.\,\text{Cos}2\alpha+\text{Sin}4\alpha}{2\text{Cos}2\alpha\,.\,\text{Cos}2\alpha+\text{Cos}2\alpha}$

$=\dfrac{\text{Sin}4\alpha\,(2\text{Cos}2\alpha+1)}{\text{Cos}2\alpha\,(2\,\text{Cos}2\alpha+1)}=\dfrac{2\,\text{Sin}\,2\alpha\,.\,\text{Cos}2\alpha}{\text{Cos}2\alpha}$

$=2\,\text{Sin}\,2\alpha$

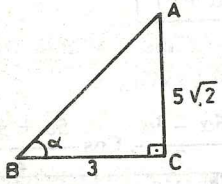
d) $\dfrac{\text{Cos}\,(\alpha+\beta)-\text{Cos}\,(\alpha-\beta)}{\text{Sin}\,(\alpha+\beta)+\text{Sin}\,(\alpha-\beta)}$

$=\dfrac{-2\,\text{Sin}\,[\frac{(\alpha+\beta+\alpha-\beta)}{2}]\,.\,\text{Sin}\,[\frac{(\alpha+\beta-\alpha+\beta)}{2}]}{2\,\text{Sin}\,[\frac{(\alpha+\beta+\alpha-\beta)}{2}]\,.\,\text{Cos}\,[\frac{(\alpha+\beta)-(\alpha-\beta)}{2}]}$

$=\dfrac{-\,\text{Sin}\,\alpha\,.\,\text{Sin}\,\beta}{\text{Sin}\,\alpha\,.\,\text{Cos}\,\beta}=-\,\text{tg}\,\beta$

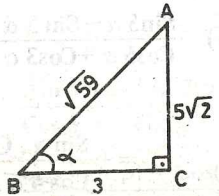
12. $0<\alpha<\dfrac{\pi}{2}$ ve $\tan\alpha=\dfrac{5\sqrt{2}}{3}$ ise, Sin 2 α ve Cos 2 α yı hesaplaymız.

ÇÖZÜM : Taslak bir dik üçgen çizerek α 'yı yerleştirelim.



Burada tan α tanımını kullanarak kenarları yazabiliriz.

Dik üçgenden $\text{Sin}\,\alpha=\dfrac{5\sqrt{2}}{\sqrt{59}}$



$\text{Cos}\,\alpha=\dfrac{3}{\sqrt{59}}$

$\left[\begin{array}{c}|AB|^2=3^2+(5\sqrt{2})^2=9+50=59\\ |AB|=\sqrt{59}\end{array}\right]$

a) $\text{Sin}\,2\alpha=2\,\text{Sin}\,\alpha\,.\,\text{Cos}\,\alpha=2\,.\,\dfrac{5\sqrt{2}}{\sqrt{59}}\,.\,\dfrac{3}{\sqrt{59}}=\dfrac{30\sqrt{2}}{59}$

b) $\text{Cos}\,2\alpha=2\,\text{Cos}^2\,\alpha-1=2\,[\dfrac{3}{\sqrt{59}}]^2-1=\dfrac{18}{59}-1$

$=-\dfrac{41}{59}$ bulunur.

13. $\widehat{ABC}$ nin açıları arasında aşağıdaki bağıntıların doğru olduğunu gösteriniz.

a) $\text{Sin}A+\text{Sin}B+\text{Sin}C=4\,\text{Cos}\dfrac{A}{2}\,.\,\text{Cos}\dfrac{B}{2}\,.\,\text{Cos}\dfrac{C}{2}$

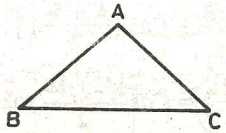
b) $\text{Cos}A+\text{Cos}B+\text{Cos}C=1+4\,\text{Sin}\dfrac{A}{2}\,.\,\text{Sin}\dfrac{B}{2}\,.\,\text{Sin}\dfrac{C}{2}$

c) $\text{Sin}A+\text{Sin}B-\text{Sin}C=4\,\text{Sin}\dfrac{A}{2}\,.\,\text{Sin}\dfrac{B}{2}\,.\,\text{Cos}\dfrac{C}{2}$

d) $\dfrac{\text{Cos}A}{\text{Sin}B\,.\,\text{Sin}C}+\dfrac{\text{Cos}B}{\text{Sin}A\,.\,\text{Sin}C}+\dfrac{\text{Cos}C}{\text{Sin}A\,.\,\text{Sin}B}=2$

ÇÖZÜM :

UYARI :



$\widehat{ABC}$ de $A+B+C=180^\circ$ ve

$\dfrac{A}{2}+\dfrac{B}{2}+\dfrac{C}{2}=90^\circ$

olur.

$\frac{B+C}{2} = 90° - \frac{A}{2}$ *veya* $\frac{A+B}{2} = 90° - \frac{C}{2}$

yazabiliriz.

$Sin\left(\frac{B+C}{2}\right) = Sin\,(90° - \frac{A}{2}) = Cos\,\frac{A}{2}$

$Sin\,\frac{A+B}{2} = Sin\,(90° - \frac{C}{2}) = Cos\,\frac{C}{2}$

olduğunu görünüz. (Tümler açı özelliği)

$\left(Sin\,\alpha = 2\,Sin\,\frac{\alpha}{2} \cdot Cos\,\frac{\alpha}{2}\right)$

a) $SinA + SinB + SinC = (SinA + SinB) + SinC$

$= 2\,Sin\,\frac{A+B}{2} \cdot Cos\,\frac{A-B}{2} + 2\,Sin\,\frac{C}{2} \cdot Cos\,\frac{C}{2}$

$= 2\,Sin\,(90 - \frac{C}{2}) \cdot Cos\,\frac{A-B}{2} + 2\,Sin\,\frac{C}{2} \cdot Cos\,\frac{C}{2}$

$[\,Sin\,\frac{A+B}{2} = Sin\,(90 - \frac{C}{2}) = Cos\,\frac{C}{2}\,]$

$= 2\,Cos\,\frac{C}{2} \cdot Cos\,\frac{A-B}{2} + 2\,Sin\,\frac{C}{2} \cdot Cos\,\frac{C}{2}$

$[\,2\,Cos\,\frac{C}{2}$ parantezine alalım.]

$= 2\,Cos\,\frac{C}{2}\,[\,Cos\,\frac{A-B}{2} + Sin\,\frac{C}{2}\,]$

$= 2\,Cos\,\frac{C}{2}\,[\,Cos\,\frac{A-B}{2} + Cos\,(\frac{A+B}{2})\,]$

$[\,Sin\,\frac{C}{2} = Sin\left(90 - \left(\frac{A+B}{2}\right)\right) = Cos\,\frac{A+B}{2}$ yazdık]

$= 2\,Cos\,\frac{C}{2}\left[2\,Cos\left[\frac{\left(\frac{A-B}{2} + \frac{A+B}{2}\right)}{2}\right] Cos\left[\frac{\left(\frac{A-B}{2}\right) - \left(\frac{A+B}{2}\right)}{2}\right]\right]$

$= 2\,Cos\,\frac{C}{2}\left[2\,Cos\,\frac{A}{2} \cdot Cos\,(-\frac{B}{2})\right]$

$= \boxed{4 \cdot Cos\,\frac{C}{2} \cdot Cos\,\frac{A}{2} \cdot Cos\,\frac{B}{2}}$ bulunur.

b) $(CosA + CosB) + CosC$

$\boxed{1 - 2\,Sin^2\,\frac{C}{2}}$

$= 2\,\boxed{Cos\,\frac{A+B}{2}} \cdot Cos\,\frac{A-B}{2} + 1 - 2\,Sin^2\,\frac{C}{2}$

$Sin\,\frac{C}{2}$

$= 1 + 2\,Sin\,\frac{C}{2}\left[Cos\,\frac{A-B}{2} - \underbrace{Sin\,\frac{C}{2}}\right]$

$\boxed{Cos\,\frac{A+B}{2}}$

$= 1 + 2\,Sin\,\frac{C}{2}\,[\,Cos\,\frac{A-B}{2} - Cos\,\frac{A+B}{2}\,]$

$= 1 + 2\,Sin\,\frac{C}{2}\left[-2\,Sin\left[\frac{\left(\frac{A-B}{2} + \frac{A+B}{2}\right)}{2}\right] Sin\left[\frac{\left(\frac{A-B}{2} - \frac{A+B}{2}\right)}{2}\right]\right]$

$$= 1 - 4 \operatorname{Sin}\frac{C}{2}\left[\operatorname{Sin}\frac{A}{2} \cdot \underbrace{\operatorname{Sin}\left(-\frac{B}{2}\right)}_{-\operatorname{Sin}\frac{B}{2}}\right]$$

$$= 1 + 4 \cdot \operatorname{Sin}\frac{A}{2} \cdot \operatorname{Sin}\frac{B}{2} \cdot \operatorname{Sin}\frac{C}{2} \quad \text{bulunur.}$$

c) $(\operatorname{Sin}A + \operatorname{Sin}B) - \underbrace{\operatorname{Sin}C}_{2\operatorname{Sin}\frac{C}{2}\operatorname{Cos}\frac{C}{2}}$

$$= 2\underbrace{\operatorname{Sin}\frac{A+B}{2}}_{\operatorname{Cos}\frac{C}{2}} \cdot \operatorname{Cos}\frac{A-B}{2} - 2\operatorname{Sin}\frac{C}{2} \cdot \operatorname{Cos}\frac{C}{2}$$

$$= 2\operatorname{Cos}\frac{C}{2} \cdot \operatorname{Cos}\frac{A-B}{2} - 2\operatorname{Sin}\frac{C}{2} \cdot \operatorname{Cos}\frac{C}{2}$$

$$= 2\operatorname{Cos}\frac{C}{2}\left[\operatorname{Cos}\frac{A-B}{2} - \underbrace{\operatorname{Sin}\frac{C}{2}}_{\operatorname{Cos}\frac{A+B}{2}}\right]$$

$$= 2\operatorname{Cos}\frac{C}{2}\left[\operatorname{Cos}\frac{A-B}{2} - \operatorname{Cos}\frac{A+B}{2}\right]$$

$$= 2\operatorname{Cos}\frac{C}{2}\left[-2\operatorname{Sin}\left(\frac{\frac{A-B}{2}+\frac{A+B}{2}}{2}\right)\operatorname{Sin}\left(\frac{\frac{A-B}{2}-\frac{A+B}{2}}{2}\right)\right]$$

$$= 2\operatorname{Cos}\frac{C}{2}\left[-2\operatorname{Sin}\frac{A}{2} \cdot \underbrace{\operatorname{Sin}\left(-\frac{B}{2}\right)}_{-\operatorname{Sin}\frac{B}{2}}\right]$$

$$= 4\operatorname{Sin}\frac{A}{2} \cdot \operatorname{Sin}\frac{B}{2} \cdot \operatorname{Cos}\frac{C}{2}$$

d) $\dfrac{\operatorname{Cos}A}{\underset{(\operatorname{Sin}A)}{\operatorname{Sin}B \cdot \operatorname{Sin}C}} + \dfrac{\operatorname{Cos}B}{\underset{(\operatorname{Sin}B)}{\operatorname{Sin}A \cdot \operatorname{Sin}C}} + \dfrac{\operatorname{Cos}C}{\underset{(\operatorname{Sin}C)}{\operatorname{Sin}A \cdot \operatorname{Sin}B}}$

$$= \frac{\operatorname{Sin}A \cdot \operatorname{Cos}A + \operatorname{Sin}B \cdot \operatorname{Cos}B + \operatorname{Sin}C \cdot \operatorname{Cos}C}{\operatorname{Sin}A \cdot \operatorname{Sin}B \cdot \operatorname{Sin}C}$$

(Payda eşitledik)

$$= \frac{2\operatorname{Sin}A \cdot \operatorname{Cos}A + 2\operatorname{Sin}B \cdot \operatorname{Cos}B + 2\operatorname{Sin}C \cdot \operatorname{Cos}C}{2\operatorname{Sin}A \cdot \operatorname{Sin}B \cdot \operatorname{Sin}C}$$

(Pay ve paydayı 2 ile çarptık)

$$= \frac{(\operatorname{Sin}2A + \operatorname{Sin}2B) + 2\operatorname{Sin}C \cdot \operatorname{Cos}C}{2\operatorname{Sin}A \cdot \operatorname{Sin}B \cdot \operatorname{Sin}C}$$

$$= \frac{2\operatorname{Sin}\frac{2A+2B}{2} \cdot \operatorname{Cos}\frac{2A-2B}{2} + 2\operatorname{Sin}C \cdot \operatorname{Cos}C}{2\operatorname{Sin}A \cdot \operatorname{Sin}B \cdot \operatorname{Sin}C}$$

$A + B = 180 - C$

$\operatorname{Sin}(A + B) = \operatorname{Sin}(180 - C)$

$= \operatorname{Sin}C$ *olduğunu hatırlayın.*

$$= \frac{2\overbrace{\operatorname{Sin}(A+B)}^{\operatorname{Sin}C} \cdot \operatorname{Cos}(A-B) + 2\operatorname{Sin}C \cdot \operatorname{Cos}C}{2\operatorname{Sin}A \cdot \operatorname{Sin}B \cdot \operatorname{Sin}C}$$

$$= \frac{\cancel{2\operatorname{Sin}C}\,[\operatorname{Cos}(A-B) + \operatorname{Cos}C]}{\cancel{2}\operatorname{Sin}A \cdot \operatorname{Sin}B \cdot \cancel{\operatorname{Sin}C}}$$

$$= \frac{\cos(A-B)+\cos C}{\sin A \cdot \sin B} = \frac{2\cos\frac{A-B+C}{2}\cdot\cos\frac{A-B-C}{2}}{\sin A \cdot \sin B}$$

$$= \frac{2\sin B \cdot \sin(B+C)}{\sin A \cdot \sin B} \quad [\, B+C = 180 - A$$

$$\sin(B+C) = \sin(180-A)$$

$$= \sin A \text{ idi. }]$$

$$= 2 \cdot \frac{\sin A}{\sin A} = 2 \quad \text{bulunur.}$$

Not :

1. $A + B + C = 180$

$A - B + C = 180 - 2B$ (Eşitliğin iki tarafından 2B çıkarıldı.)

$$\frac{A-B+C}{2} = 90 - B$$

$$\cos\left(\frac{A-B+C}{2}\right) = \cos(90-B) = \sin B$$

2. [Her iki taraftan 2B + 2C yi çıkardık.]

$A + B + C = 180$

$A - B - C = 180 - (2B + 2C)$

$$\frac{A-B-C}{2} = 90 - (B+C)$$

$$\cos\left(\frac{A-B-C}{2}\right) = \cos(90-(B+C)) = \sin(B+C)$$

14. Aşağıdaki eşitliklerin doğruluğunu gösteriniz.

a) $\dfrac{1}{\sin 10} - \dfrac{\sqrt{3}}{\cos 10} = 4$

b) $\tan A + \operatorname{Cotg} A = 2\operatorname{Cosec} 2A$

c) $\tan\left(\frac{\pi}{4} + y\right) + \tan\left(\frac{\pi}{4} - y\right) = 2\sec 2y$

d) $\dfrac{\cos 2p - \cos 4p}{\cos 2p + \cos 4p} = \tan p \cdot \tan 3p$

ÇÖZÜM :

a) $\tan 60 = \sqrt{3}$ olduğunu hatırlayınız.

$$\frac{1}{\sin 10} - \frac{\overbrace{\frac{\sin 60}{\cos 60}}^{\tan 60}}{\cos 10} = \frac{\cos 10 - \tan 60 \cdot \sin 10}{\sin 10 \cdot \cos 10}$$

$$= \frac{\cos 10 - \frac{\sin 60}{\cos 60}\cdot \sin 10}{\sin 10 \cdot \cos 10}$$

$$= \frac{\frac{\cos 10 \cdot \cos 60 - \sin 10 \cdot \sin 60}{\cos 60}}{\sin 10 \cdot \cos 10} = \frac{2[\cos(10+60)]}{\sin 10 \cdot \cos 10}$$

$$= \frac{\sin 20}{2\cos 70 \cdot \cos 10} = 2 \cdot \frac{2\sin 10 \cdot \cos 10}{\sin 20 \cdot \cos 10}$$

$$= \frac{2 \cdot (2 \cdot \sin 10 \cdot \cos 10)}{\sin 10 \cdot \cos 10} = 4$$

b) $\tan A + \operatorname{Cotg} A = \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A} = \dfrac{1}{\sin^2 A + \cos^2 A}$

$$\sin A \cdot \cos A$$

$$= \frac{1}{\text{SinA . CosA}} = \frac{2}{2\,\text{SinA . CosA}} = \frac{2}{\text{Sin 2A}}$$

$$= 2 \cdot \frac{1}{\text{Sin 2A}} = 2\,\text{Cosec 2A}$$

c) 7. bölüm 3. soruda $\text{tg}\,\alpha + \text{tg}\,\beta = \dfrac{\text{Sin}(\alpha+\beta)}{\text{Cos}\,\alpha\,.\,\text{Cos}\,\beta}$ olduğunu gördük.

$$\tan\left(\frac{\pi}{4}+y\right) + \tan\left(\frac{\pi}{4}-y\right)$$

$$= \frac{\text{Sin}\left(\left(\frac{\pi}{4}+y\right)+\left(\frac{\pi}{4}-y\right)\right)}{\text{Cos}\left(\frac{\pi}{4}+y\right)\,.\,\underbrace{\text{Cos}\left(\frac{\pi}{4}-y\right)}_{\text{Sin}\left(\frac{\pi}{4}+y\right)}} = \frac{\overbrace{\text{Sin}\,(90)}^{1}}{\text{Cos}\left(\frac{\pi}{4}+y\right).\text{Sin}\left(\frac{\pi}{4}+y\right)}$$

$$= \frac{2}{2\,\text{Cos}\left(\frac{\pi}{4}+y\right)\,.\,\text{Sin}\left(\frac{\pi}{4}+y\right)} = \frac{2}{\underbrace{\text{Sin}\,(90+2y)}_{\text{Cos}\,(-2y)}}$$

$$= \frac{2}{\text{Cos 2y}} = 2\,\text{Sec 2y} \qquad \text{Cos}\,(2y)$$

d) $$\frac{\text{Cos2p} - \text{Cos4p}}{\text{Cos2p} + \text{Cos4p}} = \frac{-\cancel{2}\,\text{Sin}\left(\frac{2p+4p}{2}\right)\,.\,\text{Sin}\left(\frac{2p-4p}{2}\right)}{\cancel{2}\,\text{Cos}\left(\frac{2p+4p}{2}\right)\,.\,\text{Cos}\left(\frac{2p-4p}{2}\right)}$$

$$= \frac{-\,\text{Sin}\,(3p)\,.\,\overbrace{\text{Sin}\,(-p)}^{(-\,\text{Sin p})}}{\text{Cos 3 p}\,.\,\underbrace{\text{Cos}\,(-p)}_{\text{Cos p}}} = \frac{\text{Sin 3p . Sin p}}{\text{Cos 3p . Cos p}}$$

$= \text{tg 3p . tg p}$ bulunur.

15. Aşağıdakileri toplam şeklinde yazınız.

a) $\text{Cos}\,4\alpha\,.\,\text{Cos}\,2\alpha$ b) $\text{Cos}\,8\alpha\,.\,\text{Cos}\,2\alpha$

c) $\text{Cos}\,10\alpha\,.\,\text{Cos}\,6\alpha$ d) $\text{Cos}\,7\alpha\,.\,\text{Cos}\,\alpha$

e) $\text{Cos}\,70\,.\,\text{Cos}\,20$ f) $\text{Cos}\,50\,.\,\text{Cos}\,40$

g) $\text{Cos}\,1\,.\,\text{Cos}\,89$ h) $\text{Sin}\,5x\,.\,\text{Sin}\,4x$

i) $\text{Sin}\,4x\,.\,\text{Cos}\,2x$ j) $\text{Sin}\,\frac{x}{2}\,.\,\text{Cos}\,\frac{3x}{2}$

k) $\text{Sin}\,35\,.\,\text{Cos}\,25$ l) $\text{Sin}\,45\,.\,\text{Cos}\,15$

m) $\text{Sin}\,(82^\circ,5)\,.\,\text{Cos}\,(37^\circ,5)$

ÇÖZÜM :

a) $$\text{Cos4}\alpha\,.\,\text{Cos2}\alpha = \frac{1}{2}\,[\,\text{Cos}\,(4\alpha+2\alpha) + \text{Cos}\,(4\alpha-2\alpha)]$$

$$= \frac{1}{2}\,[\,\text{Cos6}\,\alpha + \text{Cos2}\,\alpha\,] = \frac{\text{Cos6}\,\alpha + \text{Cos2}\,\alpha}{2}$$

(7-3 den)

b) $$\text{Cos}\,8\alpha\,.\,\text{Cos2}\,\alpha = \frac{1}{2}\,[\,\text{Cos10}\,\alpha + \text{Cos6}\,\alpha\,]$$ (7-3 den)

c) $$\text{Cos10}\,\alpha\,.\,\text{Cos6}\,\alpha = \frac{1}{2}\,[\,\text{Cos16}\,\alpha + \text{Cos4}\,\alpha\,]$$ (7-3 den)

d) $$\text{Cos}\,7\alpha\,.\,\text{Cos}\,\alpha = \frac{1}{2}\,[\,\text{Cos}\,8\alpha + \text{Cos}\,6\alpha\,]$$

e) $$\text{Cos}\,70\,.\,\text{Cos20} = \frac{1}{2}\,[\overbrace{\text{Cos90}}^{0} + \text{Cos50}] = \frac{1}{2}\,.\,\text{Cos50}$$

f) $$\text{Cos50}\,.\,\text{Cos40} = \frac{1}{2}\,[\,\text{Cos90} + \text{Cos10}\,] = \frac{1}{2}\,.\,\text{Cos10}$$

g) $\text{Cos}\,1 \cdot \text{Cos}\,89 = \frac{1}{2} \cdot \text{Cos}\,88$

h) $\text{Sin}5x \cdot \text{Sin}4x = -\frac{1}{2}[\,\text{Cos}(5x+4x) - \text{Cos}(5x-4x)]$

$= -\frac{1}{2}[\,\text{Cos}\,9x - \text{Cos}x\,]$ (7-4 den)

i) $\text{Sin}4x \cdot \text{Cos}2x = \frac{1}{2}[\,\text{Sin}(4x+2x) + \text{Sin}(4x-2x)\,]$

$= \frac{1}{2}[\,\text{Sin}6x + \text{Sin}2x\,]$ (7-1 den)

j) $\text{Sin}\frac{x}{2} \cdot \text{Cos}\frac{3x}{2} = \frac{1}{2}[\,\text{Sin}(\frac{x}{2}+\frac{3x}{2}) + \text{Sin}(\frac{x}{2}-\frac{3x}{2})\,]$

$= \frac{1}{2}[\,\text{Sin}\,2x - \text{Sin}\,x\,]$ (7-1 den)

k) $\text{Sin}35 \cdot \text{Cos}25 = \frac{1}{2}[\,\text{Sin}60 + \text{Sin}10\,]$ (7-1 den)

l) $\text{Sin}45 \cdot \text{Cos}15 = \frac{1}{2}[\,\text{Sin}60 + \text{Sin}30\,]$

$= \frac{1}{2}[\,\frac{\sqrt{3}}{2} + \frac{1}{2}\,] = \frac{\sqrt{3}+1}{4}$

m) $\text{Sin}\,(82°,5) \cdot \text{Cos}\,(37°,5) = \frac{1}{2}[\,\text{Sin}120 + \text{Sin}45\,]$

$= \frac{1}{2}[\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,] = \frac{\sqrt{3}+\sqrt{2}}{4}$

16. Aşağıdakilerin en sade yazılımlarını bulunuz.

a) $\text{Sin}\,105° + \text{Sin}\,15°$ b) $\text{Sin}\,40 + \text{Sin}\,20$

c) $\text{Cos}\,165 + \text{Cos}465$ d) $\frac{\text{Cos}\,75 + \text{Cos}\,15}{\text{Sin}\,75 - \text{Sin}\,15}$

e) $\text{Cos}\,130 + \text{Cos}\,110 + \text{Cos}\,10$

f) $\text{Cos}\,220 + \text{Cos}\,100 + \text{Cos}\,20$

ÇÖZÜM :

a) $\text{Sin}105 + \text{Sin}15 = 2\,\text{Sin}\frac{105+15}{2} \cdot \text{Cos}\frac{105-15}{2}$

$= 2\text{Sin}60 \cdot \text{Cos}45 = 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{2}$

b) $\text{Sin}40 + \text{Sin}20 = 2\,\text{Sin}\frac{40+20}{2} \cdot \text{Cos}\frac{40-20}{2}$

$= 2\,\text{Sin}30 \cdot \text{Cos}10 = 2 \cdot \frac{1}{2} \cdot \text{Cos}\,10 = \text{Cos}\,10$

c) $\text{Cos}165 + \text{Cos}465 = 2\text{Cos}\frac{165+465}{2} \cdot \text{Cos}\frac{165-465}{2}$

$= 2\,\text{Cos}315 \cdot \text{Cos}150 = 2 \cdot (+\frac{\sqrt{2}}{2}) \cdot (-\frac{\sqrt{3}}{2})$

$= -\frac{\sqrt{6}}{2}$

d) $\frac{\text{Cos}75 + \text{Cos}15}{\text{Sin}75 - \text{Sin}15} = \frac{2\,\text{Cos}45 \cdot \text{Cos}30}{2\,\text{Sin}30 \cdot \text{Cos}45} = \text{Cotg}30$

$= \sqrt{3}$

e) $(\text{Cos}130 + \text{Cos}10) + \text{Cos}110$

$= 2\,\text{Cos}\frac{130+10}{2} \cdot \text{Cos}\frac{130-10}{2} + \text{Cos}110$

$= 2\,\text{Cos}70 \cdot \text{Cos}60 + \underbrace{\text{Cos}110}_{\text{Cos}(180-70)} = 2(\text{Cos}70)\frac{1}{2} - \text{Cos}70$

$= 0$

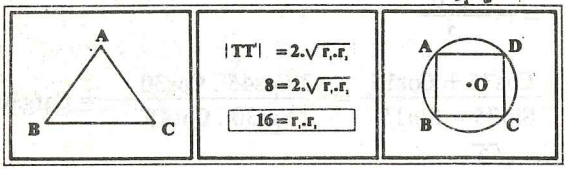
f) $(\text{Cos}220 + \text{Cos}20) + \text{Cos}100 = \overbrace{2.\text{Cos}120}^{-1/2} \cdot \text{Cos}100 + \text{Cos}100$

$= -\,\text{Cos}\,100 + \text{Cos}100 = 0$

büyük
yayınları

YENİ SINAV SİSTEMİNE GÖRE HAZIRLANMIŞTIR

ÖSS + ÖYS
için
GEOMETRİ



• 82 maddelik (şekille açıklamalı) geometri tanım-Formül Tablosu.
• Açı-üçgen-dörtgen-çember üzerine çözülmesi zorunlu tüm soru tipleri.
• Tabloya uygun, tümü yanıtlı 10 soruluk 90 kavram-öğrenme testi.
• Özellikleri ve tanımları pekiştirici tümü yanıtlı 10 soruluk 40 deneme sınavı
• Kendinizi zayıf hissettiğiniz her özelik için zengin problem literatürü.
• 1300 soruyla üniversite.

Birkan Dündar

8. BÖLÜM

TRİGONOMETRİK DENKLEMLER

(8-1) $\sin x = \sin \alpha \Rightarrow x = \alpha + k(2\pi)$ veya
$x = (\pi - \alpha) + k(2\pi)$, $k \in Z$

$$\boxed{\begin{array}{l} x = \text{İLK DEĞER} + k(2\pi) \quad \text{veya} \\ x = \text{İKİNCİ DEĞER} + k(2\pi) \end{array}}$$

(8-2) $\cos x = \cos \alpha \Rightarrow x = \alpha + k(2\pi)$ veya
$x = (2\pi - \alpha) + k(2\pi)$, $k \in Z$

$$\boxed{\begin{array}{l} x = \text{İLK DEĞER} + k(2\pi) \quad \text{veya} \\ x = \text{İKİNCİ DEĞER} + k(2\pi) \end{array}}$$

(8-3) $\operatorname{tg} x = \operatorname{tg} \alpha \Rightarrow x = \alpha + k(\pi)$, $k \in Z$

$$\boxed{x = \text{İLK DEĞER} + k(\pi)}$$

(8-4) $\operatorname{Cotg} x = \operatorname{Cotg} \alpha \Rightarrow x = \alpha + k(\pi)$, $k \in Z$

$$\boxed{x = \text{İLK DEĞER} + k(\pi)}$$

UYARI

(8-1') : $\sin x = \sin \alpha$ iken *İLK DEĞER : α*
İKİNCİ DEĞER : $\pi - \alpha$ dır.

(8-2') : $\cos x = \cos \alpha$ iken *İLK DEĞER : α*
İKİNCİ DEĞER : $2\pi - \alpha$ dır.

(8-3') : tan x = tan α iken İLK DEĞER : α dır.

(8-4') : Cotg x = Cotg α iken İLK DEĞER : α dır.

1. Aşağıdaki trigonometrik denklemlerin [0,2 π) aralığında çözüm kümesini bulunuz.

a) $\text{Sin } x = 0$ b) $\text{Sin } x = 1$ c) $\text{Sin } x = 2$

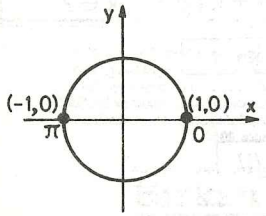
d) $\text{Sin } x = -1$ e) $\text{Sin } x = \frac{1}{2}$ f) $\text{Sin } x = \frac{\sqrt{3}}{2}$

g) $\text{Sin } x = \frac{\sqrt{2}}{2}$ h) $\text{Sin } x = -\frac{1}{2}$

i) $\text{Sin } x = -\frac{\sqrt{3}}{2}$ j) $\text{Sin } x = -\frac{\sqrt{2}}{2}$

ÇÖZÜM :

a) $\text{Sin } x = 0 \Rightarrow \text{Sin } x = \text{Sin } 0$

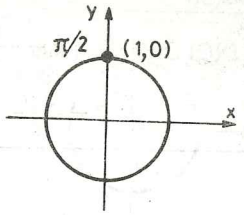


İLK DEĞER: 0
İKİNCİ DEĞER = π − 0
= π

$Ç = \{0, \pi\}$

UYARI : Birim çemberde ikinci bileşeni 0 olan 2 değer vardır, bunlar 0 ve π dir.

b) $\text{Sin } x = 1 \Rightarrow \text{Sin } x = \text{Sin } \frac{\pi}{2}$



İLK DEĞER = $\frac{\pi}{2}$

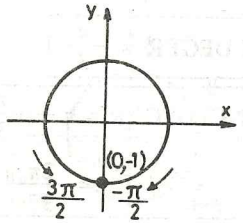
İKİNCİ DEĞER = $\pi - \frac{\pi}{2}$
= $\frac{\pi}{2}$

$Ç = \{\frac{\pi}{2}\}$

UYARI : Birim çemberde ikinci bileşeni 1 olan sadece bir değer vardır. Bu da π/2 dir.

c) $\text{Sin } x = 2$, $|\text{Sin } x| \leq 1$ idi. O halde hiçbir açının sinüsü 1 den büyük olamaz. $Ç = \phi$

d) $\text{Sin } x = -1 \Rightarrow \text{Sin } x = \text{Sin } (\frac{3\pi}{2})$



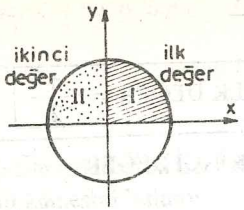
İLK DEĞER = 3π/2

İKİNCİ DEĞER = $\pi - \frac{3\pi}{2}$
= $-\frac{\pi}{2} \equiv \frac{3\pi}{2}$

$Ç = \{\frac{3\pi}{2}\}$

UYARI : Birim çemberde ikinci bileşeni — 1 olan sadece 3π/2 dir.

e) $\text{Sin } x = \frac{1}{2} \Rightarrow \text{Sin } x = \text{Sin } \frac{\pi}{6}$

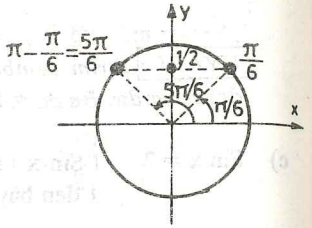


İLK DEĞER $= \frac{\pi}{6}$

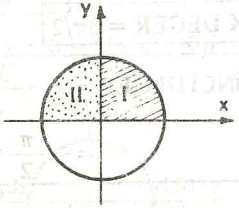
İKİNCİ DEĞER $= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

[Sinüs I. ve II. bölgede (+) dır.]

$Ç = \{\frac{\pi}{6}, \frac{5\pi}{6}\}$



f) $\text{Sin } x = \frac{\sqrt{3}}{2} \Rightarrow \text{Sin } x = \text{Sin } \frac{\pi}{3}$

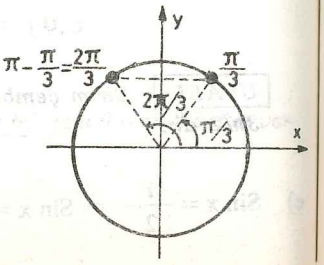


İLK DEĞER $= \frac{\pi}{3}$

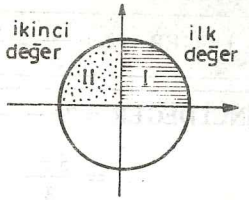
İKİNCİ DEĞER $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

[Sinüs I. ve II. bölgede (+) dır.]

$Ç = \{\frac{\pi}{3}, \frac{2\pi}{3}\}$



g) $\text{Sin } x = \frac{\sqrt{2}}{2} \Rightarrow \text{Sin } x = \text{Sin } \frac{\pi}{4}$

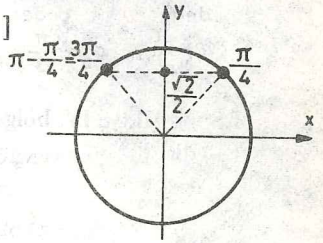


İLK DEĞER $= \frac{\pi}{4}$

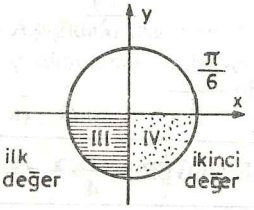
İKİNCİ DEĞER $= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

[Sinüs, I. ve II. bölgede (+) dır.]

$Ç = \{\frac{\pi}{4}, \frac{3\pi}{4}\}$



h) $\text{Sin } x = -\frac{1}{2} \Rightarrow \text{Sin } x = \text{Sin } \frac{7\pi}{6}$

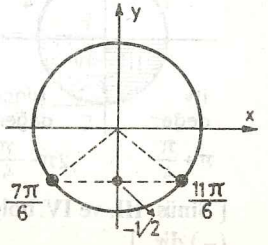


İLK DEĞER $= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

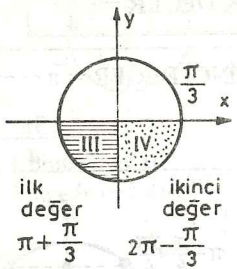
İKİNCİ DEĞER $= 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

[Sinüs III. ve IV. bölgede (−) dir.]

$Ç = \{\frac{7\pi}{6}, \frac{11\pi}{6}\}$



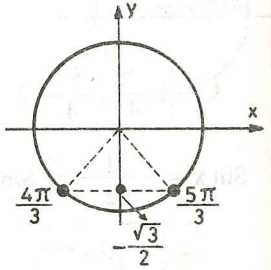
i) $\text{Sin } x = -\frac{\sqrt{3}}{2} \Rightarrow \text{Sin } x = \text{Sin } \frac{4\pi}{3}$



İLK DEĞER $= \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

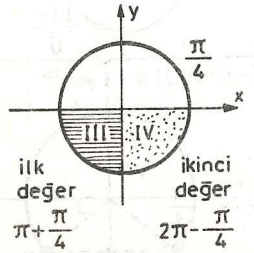
İKİNCİ DEĞER $= 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

[Sinüs III. ve IV. bölgede (–) dir.]



$\text{Ç} = \{ \frac{4\pi}{3} , \frac{5\pi}{3} \}$

j) $\text{Sin } x = -\frac{\sqrt{2}}{2} \Rightarrow \text{Sin } x = \text{Sin } \frac{5\pi}{4}$

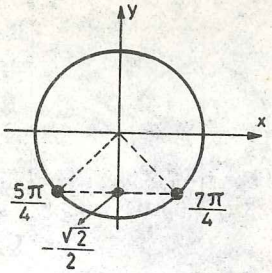


İLK DEĞER $= \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

İKİNCİ DEĞER $= 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

[Sinüs, III. ve IV. bölgede (–) dir.]

$\text{Ç} = \{ \frac{5\pi}{4} , \frac{7\pi}{4} \}$



UYARI :

(1) $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ *nin sinüs değerlerini iyi bilmeniz gerekir.*

(2) Sinüs fonksiyonunun I. ve II. bölgede (+), III. ve IV. bölgede (–) olduğunu unutmayınız.

(3) Açıların I. bölgede α *; II. bölgede* $(\pi - \alpha)$*, III. bölgede* $(\pi + \alpha)$*, IV. bölgede* $(2\pi - \alpha)$ *alındığını hatırlayınız.*

2. Aşağıdaki trigonometrik denklemlerin $[0, 2\pi)$ aralığında çözüm kümesini bulunuz.

a) $\text{Cos } x = 0$ b) $\text{Cos } x = 1$ c) $\text{Cos } x = -1$

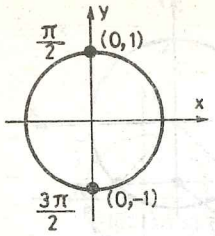
d) $\text{Cos } x = \frac{1}{2}$ e) $\text{Cos } x = \frac{\sqrt{2}}{2}$

f) $\text{Cos } x = \frac{\sqrt{3}}{2}$ g) $\text{Cos } x = -\frac{1}{2}$

h) $\text{Cos } x = -\frac{\sqrt{2}}{2}$ i) $\text{Cos } x = -\frac{\sqrt{3}}{2}$

ÇÖZÜM:

a) $\text{Cos } x = 0 \Rightarrow \text{Cos } x = \text{Cos } \frac{\pi}{2}$



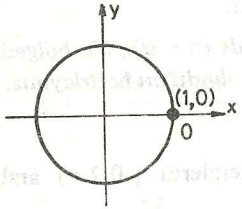
İLK DEĞER $= \frac{\pi}{2}$

İKİNCİ DEĞER $= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

$Ç = \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

UYARI : Birim çemberde birinci bileşeni 0 olan açılar π/2 ve 3π/2 dir.

b) $\text{Cos } x = 1 \Rightarrow \text{Cos } x = \text{Cos } 0$



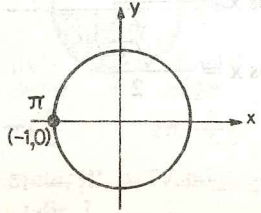
İLK DEĞER $= 0$

İKİNCİ DEĞER $= 2\pi - 0 = 2\pi \equiv 0$

$Ç = \{0\}$

UYARI : Birim çemberde birinci bileşeni 1 olan açı sadece 0 dır.

c) $\text{Cos } x = -1 \Rightarrow \text{Cos } x = \text{Cos } \pi$



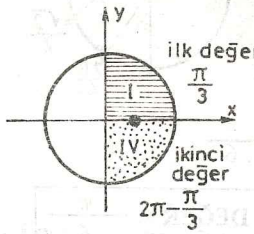
İLK DEĞER $= \pi$

İKİNCİ DEĞER $= 2\pi - \pi = \pi$

$Ç = \{\pi\}$

UYARI : Birim çemberde birinci bileşeni −1 olan açı sadece π dir.

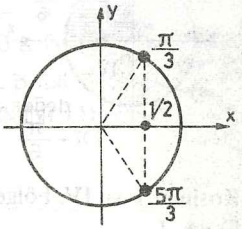
d) $\text{Cos } x = \frac{1}{2} \Rightarrow \text{Cos } x = \text{Cos } \frac{\pi}{3}$



İLK DEĞER $= \frac{\pi}{3}$

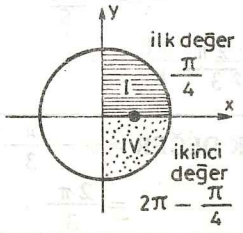
İKİNCİ DEĞER $= 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

[Kosinüs I. ve IV. bölgede (+) dır.]



$Ç = \{\frac{\pi}{3}, \frac{5\pi}{3}\}$

e) $\text{Cos } x = \frac{\sqrt{2}}{2} \Rightarrow \text{Cos } x = \text{Cos } \frac{\pi}{4}$

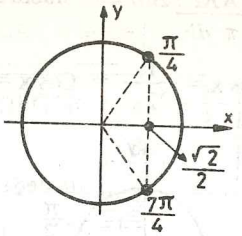


İLK DEĞER $= \frac{\pi}{4}$

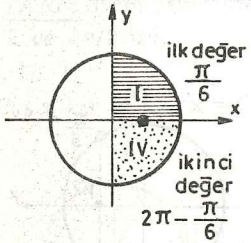
İKİNCİ DEĞER $= 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

[Kosinüs I. ve IV. bölgede (+) dır.]

$$Ç = \{\frac{\pi}{4}, \frac{7\pi}{4}\}$$



f) $\text{Cos } x = \frac{\sqrt{3}}{2} \Rightarrow \text{Cos } x = \text{Cos } \frac{\pi}{6}$

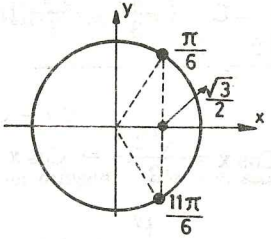


İLK DEĞER $= \frac{\pi}{6}$

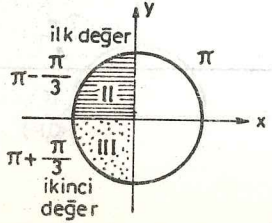
İKİNCİ DEĞER $= 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

[Kosinüs, I. ve IV. bölgede (+) dır.]

$$Ç = \{\frac{\pi}{6}, \frac{11\pi}{6}\}$$



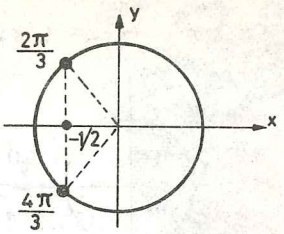
g) $\text{Cos } x = -\frac{1}{2} \Rightarrow \text{Cos } x = \text{Cos } \frac{2\pi}{3}$



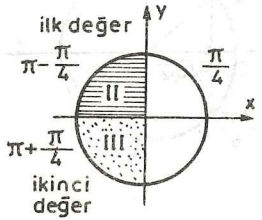
İLK DEĞER $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

İKİNCİ DEĞER $= \pi + \frac{\pi}{3}$

$$Ç = \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$$



h) $\text{Cos } x = -\frac{\sqrt{2}}{2} \Rightarrow \text{Cos } x = \text{Cos } \frac{3\pi}{4}$

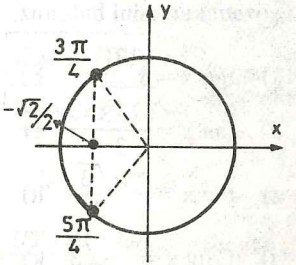


İLK DEĞER $= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

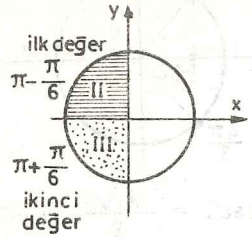
İKİNCİ DEĞER $= \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

[Kosinüs, II. ve III. bölgede (−) dir.]

$$Ç = \{\frac{3\pi}{4}, \frac{5\pi}{4}\}$$



i) $\text{Cos } x = -\frac{\sqrt{3}}{2} \Rightarrow \text{Cos } x = \text{Cos } \frac{5\pi}{6}$

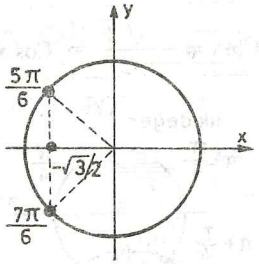


$\text{İLK DEĞER} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\text{İKİNCİ DEĞER} = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

[Kosinüs, II. ve III. bölgede (–) dir.]

$Ç = \{ \frac{5\pi}{6}, \frac{7\pi}{6} \}$



3. Aşağıdaki trigonometrik denklemlerin [0,2 π) aralığında çözüm kümesini bulunuz.

a) $\tan x = 1$ b) $\tan x = 0$ c) $\tan x = \sqrt{3}$

d) $\tan x = \frac{\sqrt{3}}{3}$ e) $\tan x = -1$ f) $\tan x = -\sqrt{3}$

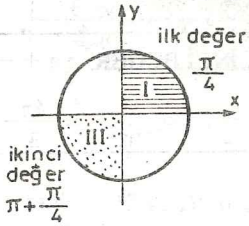
g) $\tan x = -\frac{\sqrt{3}}{3}$ h) $\text{Cotg } x = 1$ i) $\text{Cotg } x = -1$

j) $\text{Cotg } x = \sqrt{3}$ k) $\text{Cotg } x = -\sqrt{3}$

l) $\text{Cotg } x = -\frac{\sqrt{3}}{3}$

ÇÖZÜM :

a) $\tan x = 1 \Rightarrow \tan x = \tan \frac{\pi}{4}$

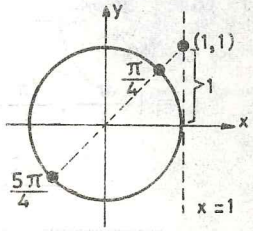


$\text{İLK DEĞER} = \frac{\pi}{4}$

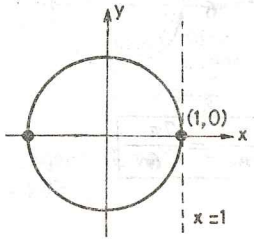
$\text{İKİNCİ DEĞER} = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

[Tanjant, I. ve III. bölgede (+) dır.]

$Ç = \{ \frac{\pi}{4}, \frac{5\pi}{4} \}$



b) $\tan x = 0 \Rightarrow \tan x = \tan 0$



$\text{İLK DEĞER} = 0$

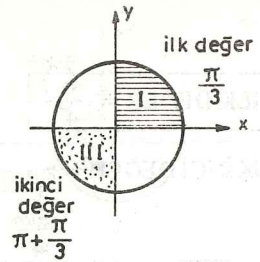
$\text{İKİNCİ DEĞER} = 0 + \pi = \pi$

$Ç = \{ 0, \pi \}$

UYARI : tanjantı 0 olan açılar 0 ve π dir.

[$\tan x = \frac{Sin\ x}{Cos\ x} = 0 \Rightarrow Sin\ x = 0 \Rightarrow x = 0\ V\ x = \pi$ *şeklinde de bulunabilir.*]

c) $\tan x = \sqrt{3} \Rightarrow \tan x = \tan \frac{\pi}{3}$

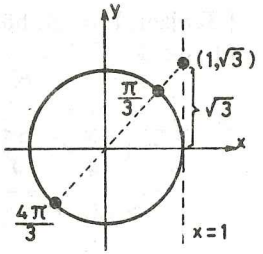


İLK DEĞER $= \frac{\pi}{3}$

İKİNCİ DEĞER $= \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

[tanjant, I. ve III. bölgede (+) dır.]

$Ç = \{\frac{\pi}{3}, \frac{4\pi}{3}\}$



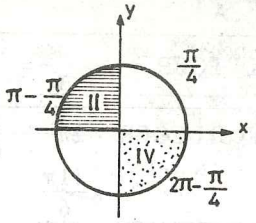
d) $\tan x = \frac{\sqrt{3}}{3} \Rightarrow \tan x = \tan \frac{\pi}{6}$

İLK DEĞER $= \frac{\pi}{6}$

İKİNCİ DEĞER $= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$Ç = \{\frac{\pi}{6}, \frac{7\pi}{6}\}$

e) $\tan x = -1 \Rightarrow \tan x = \tan \frac{3\pi}{4}$

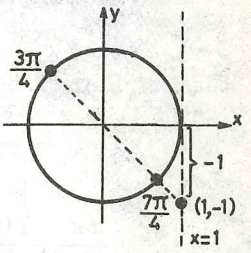


İLK DEĞER $= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

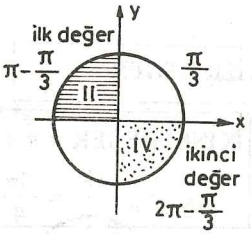
İKİNCİ DEĞER $= 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

[tanjant, II. ve IV. bölgelerde (–) dir.]

$Ç = \{\frac{3\pi}{4}, \frac{7\pi}{4}\}$



f) $\tan x = -\sqrt{3} \Rightarrow \tan x = \tan \frac{2\pi}{3}$

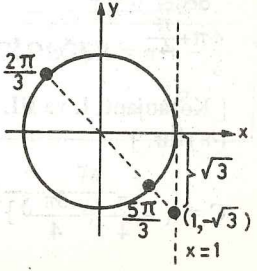


İLK DEĞER $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

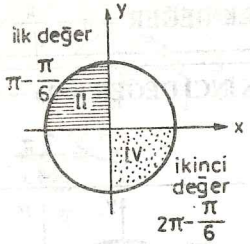
İKİNCİ DEĞER $= 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

[tanjant, II. ve IV. bölgede (–) dir.]

$Ç = \{\frac{2\pi}{3}, \frac{5\pi}{3}\}$



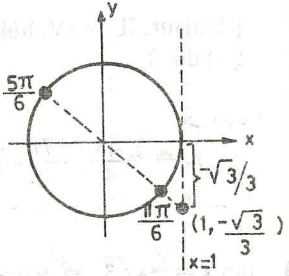
g) $\tan x = -\frac{\sqrt{3}}{3} \Rightarrow \tan x = \tan \frac{5\pi}{6}$



İLK DEĞER $= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

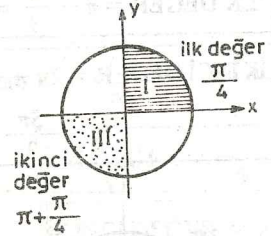
İKİNCİ DEĞER $= 2\pi - \frac{\pi}{6}$
$= \frac{11\pi}{6}$

[tanjant, II. ve IV. bölgede (–) dir.]



$Ç = \{ \frac{5\pi}{6}, \frac{11\pi}{6} \}$

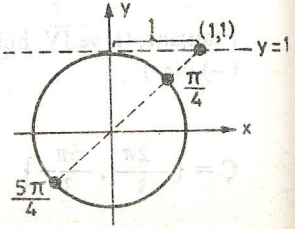
h) $\text{Cotg } x = 1 \Rightarrow \text{Cotg } x = \text{Cotg } \frac{\pi}{4}$



İLK DEĞER $= \frac{\pi}{4}$

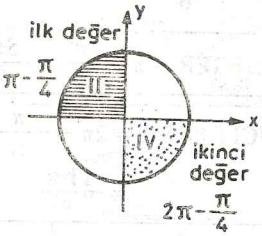
İKİNCİ DEĞER $= \pi + \frac{\pi}{4}$
$= \frac{5\pi}{4}$

[Kotanjant, I. ve III. bölgede (+) dır.]



$Ç = \{ \frac{\pi}{4}, \frac{5\pi}{4} \}$

i) $\text{Cotg } x = -1 \Rightarrow \text{Cotg } x = \text{Cotg } \frac{3\pi}{4}$

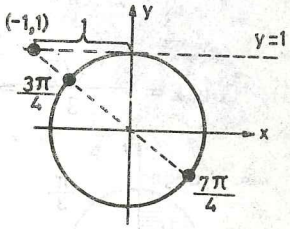


İLK DEĞER $= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

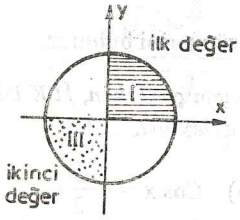
İKİNCİ DEĞER $= 2\pi - \frac{\pi}{4}$
$= \frac{7\pi}{4}$

[Kotanjant, II. ve IV. bölgede (–) dir.]

$Ç = \{ \frac{3\pi}{4}, \frac{7\pi}{4} \}$



j) $\text{Cotg } x = \sqrt{3} \Rightarrow \text{Cotg } x = \text{Cotg } \frac{\pi}{6}$

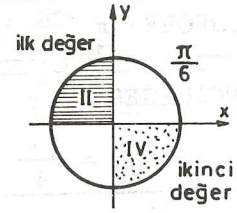


İLK DEĞER $= \frac{\pi}{6}$

İKİNCİ DEĞER $= \pi + \frac{\pi}{6}$
$= \frac{7\pi}{6}$

$Ç = \{ \frac{\pi}{6}, \frac{7\pi}{6} \}$

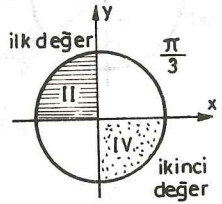
k) $\text{Cotg}x = -\sqrt{3} \Rightarrow \text{Cotg}\,x = \text{Cotg}\,(\frac{5\pi}{6})$



İLK DEĞER $= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

İKİNCİ DEĞER $= 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

l) $\text{Cotg}\,x = -\frac{\sqrt{3}}{3} \Rightarrow \text{Cotg}\,x = \text{Cotg}\,\frac{2\pi}{3}$



İLK DEĞER $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

İKİNCİ DEĞER $= 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$Ç = \{ \frac{2\pi}{3}, \frac{5\pi}{3} \}$

4. Aşağıdaki denklemlerin genel çözümlerini bulunuz.

UYARI : Sinüs ve kosinüslü denklem çözerken, İLK DEĞERE ve İKİNCİ DEĞERE k (2 π) ekleyiniz.

a) $\text{Sin}\,x = \frac{\sqrt{2}}{2}$

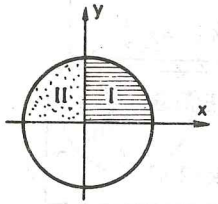
b) $\text{Cos}\,x = \frac{1}{2}$

c) $\text{Sin}\,x = -\frac{\sqrt{3}}{2}$

d $\text{Cos}\,x = -\frac{\sqrt{2}}{2}$

ÇÖZÜM :

a) $\text{Sin}\,x = \frac{\sqrt{2}}{2} \Rightarrow \text{Sin}\,x = \text{Sin}\,\frac{\pi}{4}$



İLK DEĞER $= \frac{\pi}{4} \rightarrow$

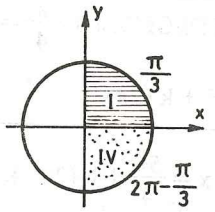
$\frac{\pi}{4} + k\,(2\,\pi)$

İKİNCİ DEĞER $= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\frac{3\pi}{4} + k\,(2\,\pi)$

$Ç = \{ x : x = \frac{\pi}{4} + k\,(2\pi)$ veya $x = \frac{3\pi}{4} + k\,(2\pi), k \in Z \}$

b) $\text{Cos}\,x = \frac{1}{2} \Rightarrow \text{Cos}\,x = \text{Cos}\,\frac{\pi}{3}$



İLK DEĞER: $\frac{\pi}{3} \rightarrow$

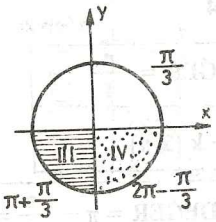
$\frac{\pi}{3} + k\,(2\,\pi)$

İKİNCİ DEĞER: $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\Rightarrow \frac{5\pi}{3} + k\,(2\,\pi)$

$Ç = \{ x : x = \frac{\pi}{3} + k\,(2\pi)$ veya $x = \frac{5\pi}{3} + k\,(2\pi), k \in Z \}$

c) $\text{Sin } x = -\frac{\sqrt{3}}{2} \Rightarrow \text{Sin } x = \text{Sin } \frac{4\pi}{3}$



İLK DEĞER: $\frac{4\pi}{3} \rightarrow$

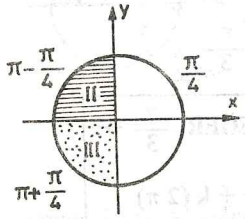
$\frac{4\pi}{3} + k(2\pi)$

İKİNCİ DEĞER: $\frac{5\pi}{3} \rightarrow$

$\frac{5\pi}{3} + k(2\pi)$

$Ç = \{ x : x = \frac{4\pi}{3} + k(2\pi) \text{ veya } x = \frac{5\pi}{3} + k(2\pi), k \in Z \}$

d) $\text{Cos } x = -\frac{\sqrt{2}}{2} \Rightarrow \text{Cos } x = \text{Cos } \frac{3\pi}{4}$



İLK DEĞER : $\pi - \frac{\pi}{4} = \frac{3\pi}{4} \rightarrow$

$\frac{3\pi}{4} + k(2\pi)$

İKİNCİ DEĞER: $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\frac{5\pi}{4} + k(2\pi)$

$Ç = \{ x : x = \frac{3\pi}{4} + k(2\pi) \text{ veya } x = \frac{5\pi}{4} + k(2\pi), k \in Z \}$

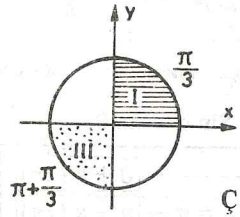
5. Aşağıdaki denklemlerin genel çözümlerini bulunuz.

UYARI : Tanjantlı ve Kotanjantlı denklemlerde İLK DEĞERE k (π) eklenir.

a) $\tan x = \sqrt{3}$ b) $\text{Cotg } x = -1$ c) $\tan x = \frac{\sqrt{3}}{3}$

ÇÖZÜM:

a) $\tan x = \sqrt{3} \Rightarrow \tan x = \tan \frac{\pi}{3}$

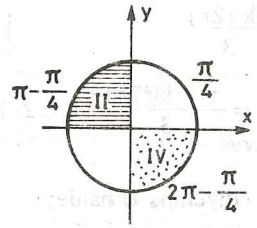


İLK DEĞER $= \frac{\pi}{3} \rightarrow$

$\frac{\pi}{3} + k(\pi)$

$Ç = \{ x : x = \frac{\pi}{3} + k(\pi), k \in Z \}$

b) $\text{Cotgx} = -1 \Rightarrow \text{Cotg } x = \text{Cotg } \frac{3\pi}{4}$

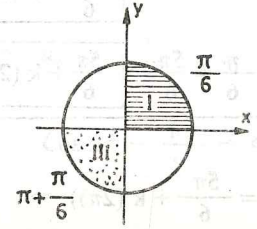


İLK DEĞER : $\frac{3\pi}{4} \rightarrow$

$\frac{3\pi}{4} + k(\pi)$

$Ç = \{ x : x = \frac{3\pi}{4} + k(\pi), k \in Z \}$

c) $\tan x = \frac{\sqrt{3}}{3} \Rightarrow \tan x = \tan \frac{\pi}{6}$



İLK DEĞER: $\frac{\pi}{6} \rightarrow \frac{\pi}{6} + k(\pi)$

$Ç = \{ x : x = \frac{\pi}{6} + k(\pi), k \in Z \}$

6. Aşağıdaki denklemlerin genel çözümlerini bulunuz.

a) $\text{Sin } 3x = 0$ b) $\text{Sin } 5x = \frac{1}{2}$

c) $\text{Cos } 7x = \frac{\sqrt{2}}{2}$ d) $\text{Cos } 8x = -\frac{1}{2}$

ÇÖZÜM :

a) $3x = \alpha$ diyelim, $\text{Sin } \alpha = 0 \Rightarrow \text{Sin } \alpha = \text{Sin } 0$

α için { İLK DEĞER : 0 $\longrightarrow$ $0 + k(2\pi)$; İKİNCİ DEĞER: $\pi - 0 = \pi \rightarrow$ $\pi + k(2\pi)$ }

Buradan;

$3x = 0 + k(2\pi)$ veya $3x = \pi + k(2\pi) \Rightarrow$

$x = \frac{k(\pi)}{3}$ veya $x = \frac{\pi}{3} + \frac{k(2\pi)}{3}$

$Ç = \{ x : x = \frac{k(2\pi)}{3}$ veya $x = \frac{\pi + k(2\pi)}{3}, k \in Z \}$ dir.

b) $\text{Sin } 5x = \frac{1}{2}$ iken $5x = \alpha$ diyelim. O halde;

$\text{Sin } \alpha = \frac{1}{2} \Rightarrow \text{Sin } \alpha = \text{Sin } \frac{\pi}{6}$

α için { İLK DEĞER $= \frac{\pi}{6} \longrightarrow \frac{\pi}{6} + k(2\pi)$; İKİNCİ DEĞER $= \pi - \frac{\pi}{6} = \frac{5\pi}{6} \rightarrow \frac{5\pi}{6} + k(2\pi)$ }

O halde;

$(5x = \frac{\pi}{6} + k(2\pi)$ veya $5x = \frac{5\pi}{6} + k(2\pi)) \Rightarrow$

$x = \frac{\pi}{30} + \frac{k(2\pi)}{5}$ veya $x = \frac{5\pi}{30} + \frac{k(2\pi)}{5}$

$Ç = \{ x : x = \frac{\pi}{30} + \frac{k(2\pi)}{5}$ veya $x = \frac{5\pi}{30} + \frac{k(2\pi)}{5}, k \in Z \}$

c) $\text{Cos } 7x = \frac{\sqrt{2}}{2}$ iken $7x = \alpha$ dersek,

$\text{Cos } \alpha = \frac{\sqrt{2}}{2} \Rightarrow \text{Cos } \alpha = \text{Cos } \frac{\pi}{4}$

α için { İLK DEĞER $= \frac{\pi}{4} \longrightarrow \frac{\pi}{4} + k(2\pi)$; İKİNCİ DEĞER $= 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \longrightarrow \frac{7\pi}{4} + k(2\pi)$ }

olur. Buradan ;

$(7x = \frac{\pi}{4} + k(2\pi)$ veya $7x = \frac{7\pi}{4} + k(2\pi) \Rightarrow$

$x = \frac{\pi + k(8\pi)}{28}$ veya $x = \frac{7\pi + k(8\pi)}{28}$ bulunur.

$Ç = \{ x : x = \frac{\pi + 8k\pi}{28}$ veya $x = \frac{7\pi + 8k\pi}{28}, k \in Z \}$

d) $\text{Cos } 8x = -\frac{1}{2}$ iken $8x = \alpha$ diyelim.

$\text{Cos } \alpha = -\frac{1}{2} \Rightarrow \text{Cos } \alpha = \text{Cos } \frac{2\pi}{3}$

α için $\begin{cases} \text{İLK DEĞER} = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \rightarrow \boxed{\frac{2\pi}{3} + k(2\pi)} \\ \text{İKİNCİ DEĞER} = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \rightarrow \boxed{\frac{4\pi}{3} + k(2\pi)} \end{cases}$

olur. Buradan;

$(8x = \frac{2\pi}{3} + k(2\pi)$ veya $8x = \frac{4\pi}{3} + k(2\pi) \Rightarrow$

$x = \frac{2\pi}{24} + \frac{k(2\pi)}{8}$ veya $x = \frac{4\pi}{24} + \frac{k(2\pi)}{8}$ dir.

$Ç = \{x : x = \frac{\pi}{12} + \frac{k\pi}{4}$ veya $x = \frac{\pi}{6} + \frac{k\pi}{4}, k \in Z\}$

7. Aşağıdaki denklemlerin genel çözümlerini bulunuz.

a) $\tan 2x = \frac{\sqrt{3}}{3}$ b) $\tan 5x = -1$

c) $\text{Cot g } x = -\sqrt{3}$

ÇÖZÜM : a) $\tan 2x = \frac{\sqrt{3}}{3}$ iken, $\boxed{2x = \alpha}$ diyelim.

$\tan \alpha = \frac{\sqrt{3}}{3} \Rightarrow \tan \alpha = \tan \frac{\pi}{6}$

α için İLK DEĞER $= \frac{\pi}{6}$ $\boxed{\frac{\pi}{6} + k\pi}$ olur.

O halde; $2x = \frac{\pi}{6} + k\pi \Rightarrow x = \frac{\pi}{12} + \frac{k\pi}{2}$ dir.

$Ç = \{x : x = \frac{\pi}{12} + \frac{k\pi}{2}, k \in Z\}$

b) $\tan 5x = -1$ ise, $\boxed{5x = \alpha}$ diyelim.

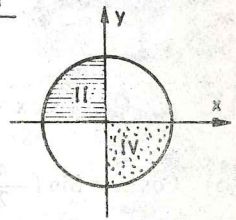
$\tan \alpha = -1 \Rightarrow \tan \alpha = \tan \frac{3\pi}{4}$

α için İLK DEĞER $= \frac{3\pi}{4}$ $\boxed{\frac{3\pi}{4} + k\pi}$ olur

$5x = \frac{3\pi}{4} + k\pi \Rightarrow x = \frac{3\pi}{20} + \frac{k\pi}{5}$ dir.

$Ç = \{x : x = \frac{3\pi}{20} + \frac{k\pi}{5}, k \in Z\}$

c) $\text{Cotg } x = -\sqrt{3} \Rightarrow \text{Cotg } x = \text{Cotg } \frac{5\pi}{6}$



İLK DEĞER : $\frac{5\pi}{6} \rightarrow \boxed{\frac{5\pi}{6} + k(\pi)}$

$x = \frac{5\pi}{6} + k(\pi)$

$Ç = \{x : x = \frac{5\pi}{6} + k(\pi), k \in Z\}$

8. Aşağıdaki denklemlerin çözüm kümesini bulunuz.

a) $\text{Cos}3x = \text{Cos } x$ b) $\text{Cos } 8x = \text{Sin}(\frac{\pi}{2} - 2x)$

c) $\text{Sin}4x = \text{Cos}(\frac{\pi}{2} - x)$

d) $\text{Cos}(x - \frac{\pi}{4}) = \text{Cos}(-x + \frac{\pi}{12})$

e) $\text{Sin}(\frac{\pi}{2} - x) = \text{Cos}(x + \pi)$

f) $\tan(3x + \frac{\pi}{8}) = \tan(x - \frac{\pi}{12})$

g) $\tan(2x - \pi) = \text{Cotg}(\frac{\pi}{2} + x)$

h) $\tan(x - \frac{\pi}{8}) \cdot \text{Cotg}2x = 1$

ÇÖZÜM :

a) $\text{Cos}3x = \text{Cos}\,x$ **İLK DEĞER** $3x = x + k(2\pi)$

$\Rightarrow 2x = k(2\pi) \Rightarrow x = k\pi$

İKİNCİ DEĞER $3x = (2\pi - x) + k(2\pi)$

$\Rightarrow 4x = k(2\pi) \Rightarrow x = \frac{k(2\pi)}{4}$

$Ç = \{ x : x = k\pi \text{ veya } x = \frac{k\pi}{2}, k \in Z \}$

) $\text{Cos}8x = \text{Sin}(\frac{\pi}{2} - 2x) \Rightarrow \text{Cos}8x = \text{Cos}(2x)$

İLK DEĞER : $8x = 2x + k(2\pi) \Rightarrow 6x = k(2\pi)$

$\Rightarrow x = \frac{k\pi}{3}$

İKİNCİ DEĞER: $8x = (2\pi - 2x) + k(2\pi) \Rightarrow 10x = k(2\pi)$

$\Rightarrow x = \frac{k\pi}{5}$

$Ç = \{ x : x = \frac{k\pi}{3} \text{ veya } x = \frac{k\pi}{5}, k \in Z \}$

c) $\text{Sin}4x = \text{Cos}(\frac{\pi}{2} - x) \Rightarrow \text{Sin}4x = \text{Sin}x$

İLK DEĞER: $4x = x + k(2\pi) \Rightarrow 3x = k(2\pi),$

$x = \frac{k(2\pi)}{3}$

İKİNCİ DEĞER: $4x = (\pi - x) + k(2\pi) \Rightarrow 5x = k(2\pi) + \pi$

$x = \frac{\pi + 2k\pi}{5}$

$Ç = \{ x : x = \frac{\pi + 2k\pi}{5} \text{ veya } x = \frac{k(2\pi)}{3}, k \in Z \}$

d) $\text{Cos}(x - \frac{\pi}{4}) = \text{Cos}(-x + \frac{\pi}{12})$

İLK DEĞER : $x - \frac{\pi}{4} = -x + \frac{\pi}{12} + k(2\pi) \Rightarrow$

$2x = \frac{\pi}{12} + \frac{\pi}{4} + k(2\pi) \Rightarrow$

$x = \frac{4\pi + k(24\pi)}{24} = \frac{4(\pi + 6k\pi)}{24}$

$= \frac{\pi + 6k\pi}{6}$

İKİNCİ DEĞER: $x - \frac{\pi}{4} = 2\pi - [-x + \frac{\pi}{12}] + k(2\pi)$

$x - \frac{\pi}{4} = 2\pi + x - \frac{\pi}{12} + k(2\pi)$

$0 = \frac{\pi}{4} + 2\pi + \frac{\pi}{12} + k(2\pi)$ (çözüm yok)

$Ç = \{ x : x = \frac{\pi}{6} + k\pi, k \in Z \}$

e) $\text{Sin}\left(\frac{\pi}{2} - x\right) = \text{Cos}(x + \pi) \Rightarrow \text{Cos}\, x = \text{Cos}(x + \pi)$

İLK DEĞER : $x = x + \pi + 2k\pi$; $0 = \pi + 2k\pi$ (çözüm yok)

İKİNCİ DEĞER: $x = [2\pi - (x + \pi)] + k2\pi$; $2x = \pi + k(2\pi)$

$$Ç = \{x : x = \frac{\pi}{2} + k\pi,\ (k \in Z)\}$$

f) $\tan\left(3x + \frac{\pi}{8}\right) = \tan\left(x - \frac{\pi}{12}\right) \Rightarrow 3x + \frac{\pi}{8} = x - \frac{\pi}{12} + k(\pi)$

$$2x = -\frac{\pi}{8} - \frac{\pi}{12} + k\pi \Rightarrow 2x = \frac{-5\pi}{24} + k\pi \Rightarrow$$

(3) (2)

$$x = \frac{-5\pi}{48} + \frac{k\pi}{2}$$

$$Ç = \{x : x = \frac{-5\pi}{48} + \frac{k\pi}{2},\ k \in Z\}$$

g) $\tan(2x - \pi) = \text{Cotg}\left(\frac{\pi}{2} + x\right) \Rightarrow \tan(2x - \pi) = \tan(-x)$

$$2x - \pi = -x + k\pi \Rightarrow 3x = \pi + k\pi \Rightarrow 3x = k\pi \Rightarrow x = \frac{k\pi}{3}$$

$$Ç = \{x : x = \frac{k\pi}{3},\ k \in Z\}$$

h) $\tan\left(x - \frac{\pi}{8}\right) \cdot \text{Cotg}\, 2x = 1 \Rightarrow \tan\left(x - \frac{\pi}{8}\right) = \frac{1}{\text{Cotg}\, 2x}$

$$\Rightarrow \tan\left(x - \frac{\pi}{8}\right) = \tan 2x$$

İLK DEĞER : $x - \frac{\pi}{8} = 2x + k\pi$; $x = k\pi - \frac{\pi}{8}$

$$Ç = \{x : x = k\pi - \frac{\pi}{8},\ k \in Z\}$$

8. Aşağıdaki denklemlerin $[0, 2\pi)$ aralığında çözüm kümelerini bulunuz.

a) $\text{Cos}^2 x = \frac{1}{2}$ b) $\text{Sin}\, x \cdot \text{Cos}\, x = 0$

c) $\text{Sin}2x + \text{Sin}x = 0$ d) $\text{Cos}x + \text{Cos}2x = 0$

e) $\text{Sin}^2 x + \text{Sin}x - 2 = 0$ f) $3\,\text{Cos}^2 x - \text{Sin}^2 x = 0$

ÇÖZÜM :

a) $\text{Cos}^2 x = \frac{1}{2} \Rightarrow \sqrt{\text{Cos}^2 x} = \sqrt{\frac{1}{2}} \Rightarrow |\text{Cos}\, x| = \frac{1}{\sqrt{2}}$

$\text{Cos}\, x = \frac{1}{\sqrt{2}}$ veya $\text{Cos}\, x = -\frac{1}{\sqrt{2}}$ dir.

$$Ç = \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$$

b) $\text{Sin}\, x \cdot \text{Cos}\, x = 0 \Rightarrow \text{Sin}\, x = 0$ veya $\text{Cos}\, x = 0$ dır.

$$Ç = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$$

c) $\text{Sin}2x + \text{Sin}\, x = 0 \Rightarrow 2\,\text{Sin}\, x \cdot \text{Cos}\, x + \text{Sin}\, x = 0 \Rightarrow$

$\text{Sin}\, x\,(2\text{Cos}\, x + 1) = 0 \Rightarrow \text{Sin}\, x = 0$ veya $\text{Cos}x = -\frac{1}{2}$

dir. $Ç = \{0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}\}$

d) $\text{Cos}\,x + \text{Cos}\,2x = 0 \Rightarrow \text{Cos}\,x + 2\text{Cos}^2x - 1 = 0$,
(Cos x = t dersek, 2. derece denklem çözümüne indirgeriz.)

$$2t^2 + t - 1 = 0 \Rightarrow \begin{matrix} t_1 = -1 \\ t_2 = \frac{1}{2} \end{matrix} \Bigg\} \quad \text{Cosx} = -1 \text{ veya } \text{Cosx} = \frac{1}{2}$$

$$Ç = \{ \frac{\pi}{3}, \frac{5\pi}{3}, \pi \}$$

e) $\text{Sin}^2x + \text{Sin}\,x - 2 = 0$; Sin x = t dersek ;

$$t^2 + t - 2 = 0 \Rightarrow \begin{matrix} t_1 = 1 \\ t_2 = -2 \end{matrix}$$

Sin x = 1 veya Sin x = – 2 olur. [Sin x ≠ – 2]

$$Ç = \{ \frac{\pi}{2} \}$$

f) $3\,\text{Cos}^2x - \text{Sin}^2x = 0 \Rightarrow 3\,\text{Cos}^2x = \text{Sin}^2x \Rightarrow 3 = \frac{\text{Sin}^2x}{\text{Cos}^2x}$

$\Rightarrow \tan^2x = 3 \Rightarrow \tan x = \sqrt{3}$ veya $\tan x = -\sqrt{3}$ dür.

$$Ç = \{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \}$$

9. Aşağıdaki denklemlerin [0,2 π) aralığında çözüm kümelerini yazınız.

a) $2\,\text{Sin}\,x = 1 + \text{Cosec}\,x$ b) $2\text{Sec}\,x = \tan x + \text{Cotg}x$

c) $\text{Sec}\,x + \tan x = 0$ d) $\text{Sin}\,x + \text{Cos}\,x = 1$

ÇÖZÜM :

a) $2\,\text{Sin}\,x = 1 + \text{Cosec}\,x \Rightarrow 2\,\text{Sin}\,x = 1 + \frac{1}{\text{Sin}\,x}$

$\Rightarrow 2\,\text{Sin}^2x = \text{Sin}\,x + 1 \Rightarrow 2\,\text{Sin}^2x - \text{Sin}\,x - 1 = 0$;

Sin x = t dersek ;

$$2t^2 - t - 1 = 0 \Rightarrow \begin{matrix} t_1 = -1/2 \\ t_2 = 1 \end{matrix} \Rightarrow \text{Sin}\,x = -\frac{1}{2} \text{ veya}$$

Sin x = 1 dir.

$$Ç = \{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \}$$

b) $2\,\text{Sec}\,x = \tan x + \text{Cotg}\,x \Rightarrow \frac{2}{\text{Cosx}} = \frac{\text{Sinx}}{\text{Cosx}} + \frac{\text{Cosx}}{\text{Sinx}}$

$$\Rightarrow \frac{2}{\cancel{\text{Cos}\,x}} = \frac{1}{\text{Sinx}\,.\,\cancel{\text{Cosx}}} \Rightarrow [\,\text{Cos}\,x \neq 0\,]$$

$$\text{Sin}\,x = \frac{1}{2} \text{ dir.} \qquad Ç = \{ \frac{\pi}{6}, \frac{5\pi}{6} \}$$

c) $\text{Sec}\,x + \tan x = 0 \Rightarrow \frac{1}{\text{Cos}\,x} + \frac{\text{Sin}\,x}{\text{Cos}\,x} = 0 \Rightarrow 1 + \text{Sin}x = 0$

$$\Rightarrow \text{Sin}\,x = -1 \Rightarrow x = \frac{3\pi}{2}$$

[3π/2 değeri için Sec x ve tan x tanımlı değildir.]

$$\boxed{Ç = \phi}$$

d) $\text{Sin}\,x + \text{Cos}\,x = 1 \Rightarrow (\text{Sin}\,x + \text{Cos}\,x)^2 = 1 \Rightarrow$

$$\underbrace{\text{Sin}^2x + 2\,\text{Sin}\,x\,.\,\text{Cos}\,x + \text{Cos}^2x}_{1} = 1$$

$1 + \text{Sin}2x = 1 \Rightarrow \text{Sin}2x = 0 \Rightarrow x = 0$ $\qquad Ç = \{ 0, \frac{\pi}{2} \}$

$2x = 0$ $\qquad x = \frac{\pi}{2}$

$2x = 0 + 2\pi$

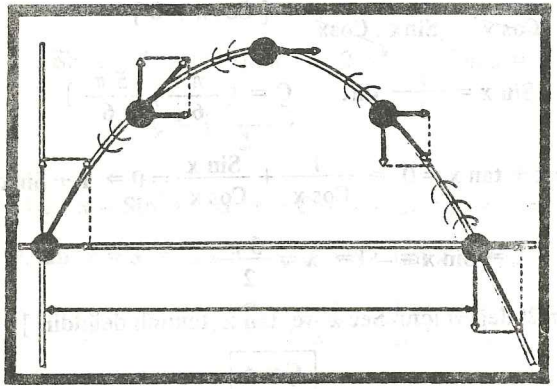
$2x = \pi$

$2x = \pi + 2\pi$

büyük
yayınları

YENİ
PROGRAMA UYGUN

Lise 1
FİZİK



• Konu Açıklamalı
• Örnek Çözümlü
• Cevaplı Sorular

NURETTİN CERAN · NECATİ TANRIÖVER

9. BÖLÜM

TEST 01

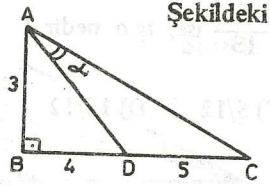
1. Sin170°, Sec100°, Cosec220°, tg279° nin işaretleri sırasıyla nedir ?

A) +, +, −, +
B) +, −, −, +
C) +, −, −, −
D) −, +, −, +
E) +, +, +, −

2. Bir üçgenin kenarları 3, 4 ve 5 dir. Bu üçgende en küçük açının kosinüsü nedir ?

A) 3/5 B) 4/5 C) 3/8 D) 1/2 E) 3/4

3. Şekildeki verilere göre, tan α nedir



A) 1/3
B) 4/5
C) 3
D) 2
E) 1

4. θ açısı 90° den 180° ye artarken Cos θ nasıl değişir ?

A) 0 dan − 1'e azalır.
B) − 1 den 0'a artar.
C) 0 dan 1'e artar.
D) − 1 den 1'e artar.
E) 1 den 0'a azalır.

5. Sin $\frac{147\pi}{4}$ ün eşiti nedir ?

A) $\frac{\sqrt{2}}{2}$ B) $-\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{2}$

D) $-\frac{\sqrt{3}}{2}$ E) 1

6. $(\text{Sin}\,\alpha + \text{Cos}\,\alpha)^2 - \text{Sin}2\alpha = ?$

A) 1 B) -1 C) 0 D) Sin α E) Sin α . Cos α

7. $1 + \text{Cos}\,7\alpha$ ifadesinin en büyük değeri kaçtır ?

A) 8 B) 7 C) 4 D) 2 E) 1

8. $0 < \alpha < \frac{\pi}{2}$ iken $\text{Sin}\,\alpha = \frac{5}{13}$ ise, tg α nedir ?

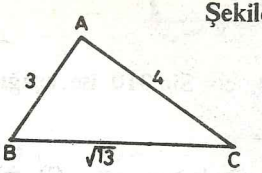
A) 12/5 B) 12/13 C) 5/12 D) 13/12 E) 13/5

9. $\alpha = \frac{\pi}{8}$ ise, $2\,\text{Cos}^2\,\alpha - 1$ kaçtır ?

A) $\frac{\sqrt{3}}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{2}}{2}$

D) $-\frac{\sqrt{2}}{2}$ E) -1

10. Şekilde m$\hat{A}$ kaç derecedir ?



A) 30° B) 45°
C) 60° D) 75°
E) 90°

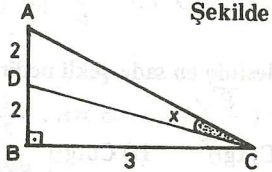
11. $\alpha \in (0, 2\pi)$ için $\text{Sin}2\alpha = \frac{1}{2}$ denkleminin çözüm kümesi aşağıdakilerden hangisidir ?

A) $\{15°\}$ B) $\{15°, 75°\}$
C) $\{15°, 75°, 195°\}$ D) $\{15°, 75°, 195°, 255°\}$
E) $\{15°, 165°\}$

12. $\alpha \in [0, 2\pi)$ için $\text{Cos}6\alpha = \frac{\sqrt{3}}{2}$ denkleminin kaç tane kökü vardır ?

A) 2 B) 4 C) 6 D) 8 E) 12

13. Şekilde Cos x kaçtır ?



A) $\frac{\sqrt{13}}{65}$ B) $\frac{7\sqrt{13}}{5}$

C) $\frac{17\sqrt{13}}{65}$ D) $\frac{7\sqrt{13}}{65}$ E) $\frac{17}{65}$

1-C 2-B 3-A 4-A 5-A 6-A 7-D
8-C 9-C 10-C 11-D 12-E 13-C

TEST 02

1. a = Sin 200, b = Sin 240, c = Sin 310 ise, aşağıdakilerden hangisi doğrudur ?

A) a > c > b B) a > b > c C) b > a > c
D) c > b > a E) c > a > b

2. $\text{Sin}x - \text{Cos}\,x = \frac{1}{\sqrt{3}}$ ise, Cos2x kaç olabilir ?

A) $\frac{\sqrt{5}}{3}$ B) $\frac{\sqrt{5}}{2}$ C) $\frac{2}{3}$ D) $\frac{1}{2}$ E) $\frac{1}{3}$

3. $\text{Sin}(91\pi + \theta) + \text{Sin}(-100\pi + \theta) + \text{Cos}(\frac{33\pi}{2} - \theta)$ ifadesinin eşiti nedir ?

A) Sin (2 θ) B) 2 Sin θ C) 3 Sin θ
D) Sin θ E) Cos θ

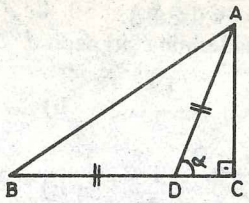
4. $\frac{1}{\text{Sin}\theta + \text{Cos}\theta \,.\, \text{Cotg}\theta}$ ifadesinin en sade şekli nedir ?

A) Sinθ B) Cosθ C) tgθ D) Cotgθ E) Secθ

5. $\alpha = \frac{5\pi}{12}$ ise, $\frac{1}{\text{tg}\,\alpha + \text{Cotg}\,\alpha}$ ifadesinin eşiti nedir ?

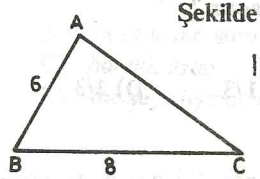
A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{1}{4}$

6. Şekilde | AD | = | BD | dir.
$\tan\alpha = \frac{3}{4}$ ise, $\text{Cotg}(\frac{\alpha}{2})$ kaçtır ?



A) 1/3 B) 3 C) 2
D) 1/2 E) 4

7. Şekilde B dar açı ve $\text{Sin}B = \frac{\sqrt{5}}{3}$ ise, | AC | = b kaç birimdir ?



A) 4 B) 5 C) 6
D) 7 E) 8

8. $\frac{\text{Cos}\,\alpha + \text{Sin}\,\alpha}{\text{Cos}\,\alpha - \text{Sin}\,\alpha}$ ifadesinin eşiti nedir ?

A) tg α B) $\text{tg}(\frac{\pi}{3} + \alpha)$ C) $\text{tg}(\frac{\pi}{4} + \alpha)$
D) $\text{tg}(\frac{\pi}{4} - \alpha)$ E) $\text{tg}(\frac{\pi}{3} - \alpha)$

9. $\frac{1}{5}$. Arc Sin $(-\frac{\sqrt{3}}{2})$ + Arc Cotg (1) ifadesinin eşiti nedir ?

A) $\frac{\pi}{12}$ B) $\frac{7\pi}{12}$ C) $\frac{5\pi}{12}$
D) $\frac{23\pi}{12}$ E) $\frac{\pi}{6}$

10. $\overset{\frown}{ABC}$ de $m\hat{A} = 30°$ ise, (SinB . Sin C – Cos B . Cos C) ifadesinin eşiti nedir ?

A) $\frac{\sqrt{3}}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $\frac{1}{2}$

D) $-\frac{1}{2}$ E) $\frac{\sqrt{2}}{2}$

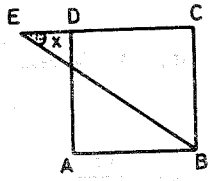
11. Arc tan $\frac{3}{4} = x$ ise, Sinx kaçtır ?

A) 3/5 B) 4/5 C) 1/3 D) 2/3 E) 3/4

12. $\frac{\pi}{2} \leq x \leq \pi$ iken $2Sin^2x - 7Sinx + 3 = 0$ denkleminin kökü aşağıdakilerden hangisidir ?

A) $\frac{4\pi}{5}$ B) $\frac{6\pi}{7}$ C) $\frac{2\pi}{3}$

D) $\frac{3\pi}{4}$ E) $\frac{5\pi}{6}$

13. 

Şekilde ABCD kare ve $|DC| = 3 . |ED|$ dir. Sec x nedir ?

A) 3/4 B) 4/5 C) 4/3
D) 5/4 E) 5/3

1-A 2-A 3-D 4-A 5-E 6-B 7-C
8-C 9-B 10-A 11-A 12-E 13-D

TEST 03

1. tg x = 2 ise, tg2x kaçtır ?

A) – 4/3 B) 4/3 C) 3/4 D) – 3/4 E) 4/5

2. θ açısı 90° den 270° ye artarken Sin θ nasıl değişir ?

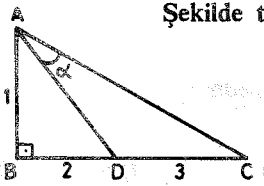
A) Önce artar, sonra azalır. B) Önce azalır, sonra artar.
C) Sürekli artar. D) Sürekli azalır.
E) Bir şey söylenemez.

3. $\frac{2}{\tan(\frac{x}{2}) + Cotg(\frac{x}{2})}$ ifadesinin eşiti nedir ?

A) Sin2x B) Cos2x C) Sinx D) Cosx E) Secx

4. $\overset{\frown}{ABC}$ de $Sin\hat{A} = Cos\hat{B}$ ise, $Cos\hat{C}$ kaçtır ?

A) 1 B) – 1 C) 0 D) $\frac{\sqrt{2}}{2}$ E) $-\frac{\sqrt{2}}{2}$

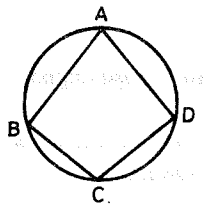
5. 

Şekilde tan α kaçtır ?

A) 1/5 B) 3/10
C) 1/11 D) 2/11
E) 3/11

6. $\overset{\triangle}{ABC}$ de $\text{Cos}(A+B)+\text{Cos}C$ nin eşiti nedir ?

A) 0 B) 2CosA C) 2CosC D) 1 E) -1

7. 

ABCD kirişler dörtgenidir.
CosA + CosB + CosC + CosD kaç tır ?

A) 4CosA B) Sin2A
C) tgA D) Sin (A + B)
E) 0

8. $\text{Cos}\frac{3\pi}{5}+\text{Cos}\frac{\pi}{5}$ in eşiti kaçtır ?

A) 1/2 B) 1/4 C) 1/6 D) 1/8 E) 1/10

9. $\frac{\text{Cos}\,8\alpha+\text{Cos}\,2\alpha}{\text{Sin}\,8\alpha+\text{Sin}\,2\alpha}$ nin eşiti nedir ?

A) tg5 α B) Cotg5 α C) tg3 α D) Cotg3α E) 2Sinα

10. Arc Sin $(-\frac{\sqrt{3}}{2})$ nin eşiti nedir ?

A) $\pi/3$ B) $2\pi/3$ C) $4\pi/3$ D) $5\pi/3$ E) $11\pi/6$

11. $\alpha=\frac{\pi}{10}$ ise, $\frac{\text{Cos}5\alpha+\text{Cos}\,\alpha}{\text{Cos}7\alpha\cdot\text{Cos}2\alpha}=?$

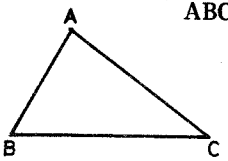
A) 2 B) -2 C) 3 D) 0 E) -1

12. $\text{Sin}\,x-\text{Cos}\,x=\frac{1}{3}$ ise, Sin2x kaçtır ?

A) 1/8 B) 1/9 C) 2/3 D) 10/9 E) 8/9

13. $\overset{\triangle}{ABC}$ nin kenarları a, b, c dir. $m\hat{A}\neq m\hat{B}$ iken, $a^3-b^3=ac^2-c^2b$ ise, $m\hat{C}$ kaç derecedir ?

A) 30° B) 45° C) 60° D) 90° E) 120°

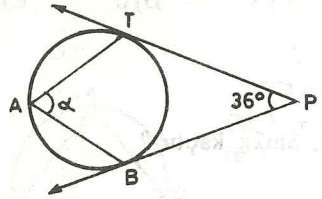
14. 

$\overset{\triangle}{ABC}$ de $\tan\hat{B}=3$, $\tan\hat{C}=2$ dir.
$m\hat{B}+m\hat{C}$ kaç derecedir ?

A) 30 B) 45 C) 60
D) 120 E) 135

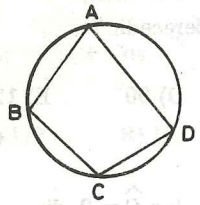
1-A 2-D 3-C 4-C 5-E 6-A 7-E
8-A 9-B 10-D 11-B 12-E 13-C 14-E

TEST 04

1. 

[PT ve [PB çembere teğet ve Cos36 = m ise, Cosα nedir ?

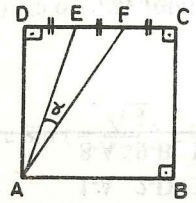
A) $2m^2 - 1$ B) $1 - 2m^2$
C) 2m D) 2m + 1
E) 1 – 2m

2. 

ABCD kirişler dörtgenidir. tanA = 2 ise, CosC nedir ?

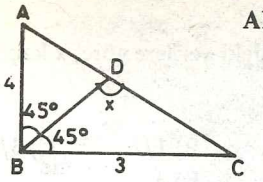
A) $\frac{\sqrt{5}}{5}$ B) $-\frac{\sqrt{5}}{5}$ C) $\frac{2}{\sqrt{5}}$

D) $-\frac{2}{\sqrt{5}}$ E) – 2

3. 

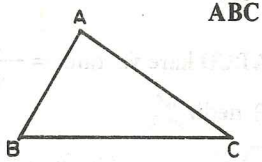
□ ABCD kare ve |DE| = |EF| = |FC| dir. tan α nedir ?

A) 3/11 B) 1/7 C) 2/7
D) 2/11 E) 2/15

4. 

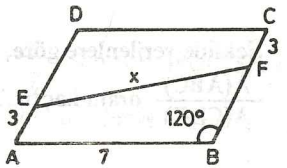
ABC dik üçgeninde verilenlere göre, tan x kaçtır ?

A) 3 B) 4 C) 5
D) 6 E) 7

5. 

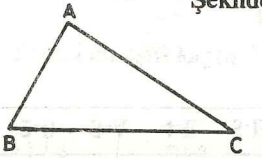
$\widehat{ABC}$ de SinB = CosC için SinA kaçtır ?

A) 0 B) 1 C) – 1
D) $\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{3}}{2}$

6. 

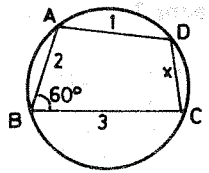
Şekilde ABCD eşkenar dörtgen ise, |EF| = x kaç birimdir ?

A) $\sqrt{57}$ B) $2\sqrt{10}$
C) $\sqrt{10}$ D) $\sqrt{19}$
E) 6

7. 

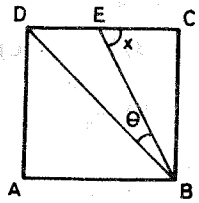
Şekilde tanB = 2, tanC = $\frac{1}{3}$ ise, tanA nedir ?

A) 7 B) – 5 C) – 6
D) – 7 E) – 8

8. 

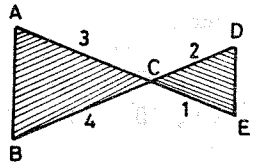
Şekildeki verilere göre, x kaç birimdir ?

A) 1 B) 1/2 C) 3/2
D) 2 E) 3

9. 

Şekilde ABCD kare ve $\tan x = \frac{4}{3}$ ise, $\tan \theta$ nedir ?

A) 1/3 B) 1/4 C) 1/5
D) 1/6 E) 1/7

10. 

Şekilde verilenlere göre, $\frac{A(ABC)}{A(CDE)}$ oranı kaçtır ?

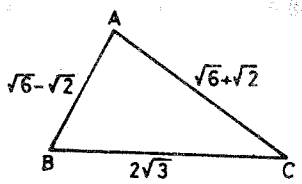
A) 2 B) 3 C) 4 D) 6 E) 36

1-A	2-B	3-A	4-E	5-B	6-A	7-D	8-D	9-E	10-D

TEST 05

1. $\frac{\text{Sin } 220}{\text{Sin } 40} + \frac{\text{Cos } (-50)}{\text{Sin } 40} + \frac{\text{tg } 230}{\text{Cotg } 40} = ?$

A) 3 B) 2 C) 1 D) – 1 E) – 2

2. 

Şekildeki verilere göre, $m\hat{A}$ kaç derecedir ?

A) 15° B) 30° C) 45°
D) 60° E) 120°

3. $\frac{\pi}{2} < \alpha < \pi$ iken, $\text{Sin } \alpha = \frac{3}{5}$ ise,
$\text{Sin}^3 \alpha \cdot \text{Cos } \alpha + \text{Cos}^3 \alpha \cdot \text{Sin} \alpha = ?$

A) 4/5 B) – 4/5 C) 12/5 D) – 12/5 E) – 12/25

4. $96\sqrt{3} \cdot \text{Sin}\frac{\pi}{48} \cdot \text{Cos}\frac{\pi}{48} \cdot \text{Cos}\frac{\pi}{24} \cdot \text{Cos}\frac{\pi}{12} \cdot \text{Cos}\frac{\pi}{6}$
ifadesinin eşiti kaçtır ?

A) 18 B) 9 C) 3 D) 1 E) 1/3

5. Cos (Arc Cotg 1) = ?

A) 0 B) 1/2 C) 1 D) $\sqrt{2}/2$ E) $\sqrt{3}/2$

6. $\frac{\text{Sin}\,(90+x)}{\text{Sin}\,(-x)} = ?$

A) Cotg x B) – Cotg x C) 4x D) – tg x E) 1

7. $\text{Cotg}\,x - \frac{\text{Cos2x}}{\text{Sinx . Cosx}} = ?$

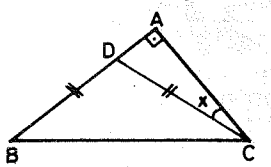
A) Cosx B) Sin2x C) Sinx D) Cotg x E) tan x

8. $\frac{c^2+a^2-b^2}{c^2+b^2-a^2} = ?$

A) $\text{tg}\,(\frac{A}{B})$ B) $\frac{tgA}{tgB}$ C) $\frac{tgB}{tgA}$ D) $\frac{SinA}{SinB}$ E) $\frac{SinB}{SinA}$

9. $\frac{\text{Sin 3x}}{\text{Sin x}} - \frac{\text{Cos3x}}{\text{Cos x}} = ?$

A) 2 B) – 2 C) 1 D) – 1 E) Sinx

10. 

$\hat{ABC}$ de |BD| = |DC| dir.

$\text{tanx} = \frac{8}{15}$ ise, tanB kaçtır ?

A) 3/4 B) 3/5 C) 17/15 D) 15/17 E) 5/4

11. $\text{Sinx} + \text{Cosx} = \frac{1}{5}$ ise, Sin2x kaçtır ?

A) – 4/5 B) 4/5 C) 24/25 D) – 24/25 E) 12/25

12. Sin7 = a ise, Sin76 kaçtır ?

A) $2a^2 - 1$ B) $1 - 2a^2$ C) 2a D) 2a + 1 E) $2a^2$

13. $\frac{\text{Sin } 155^\circ + \text{Sin } (-205)}{\text{Cos } 295} = ?$

A) 2 B) 1 C) 0 D) – 1 E) – 2

1-C 2-D 3-E 4-B 5-D 6-B 7-E
8-B 9-A 10-B 11-D 12-B 13-A

TEST 06

1. $\left.\begin{array}{l} a = \text{Sin}12^\circ \\ b = \text{Sin}88^\circ \\ c = \text{Sin}109^\circ \end{array}\right\}$ olduğuna göre, aşağıdakilerden hangisi doğrudur ?

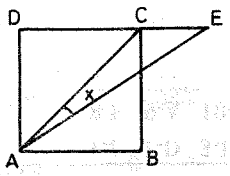
A) $a < b < c$ B) $c < b < a$ C) $a < c < b$
D) $b < c < a$ E) $b < a < c$

2. Cos210 nun değeri aşağıdakilerden hangisidir ?

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) $-\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{2}}{2}$

3. $0 < \theta < \frac{\pi}{2}$ ve $\tan\theta = \frac{1}{2}$ ise, $\cos\theta$ nın değeri kaçtır ?

A) 1 B) $\frac{\sqrt{3}}{2}$ C) $\frac{2\sqrt{5}}{5}$ D) $\frac{1}{2}$ E) $\frac{\sqrt{5}}{5}$

4. 

ABCD kare ve $|DC| = 2\,|CE|$ ise, tg x kaçtır ?

A) 1/4 B) 1/5 C) 1/6
D) 1/7 E) 1/8

5. $\text{tg}\,x \cdot (1 - \text{Cos}\,2y) = \text{Sin}\,2y$ olduğuna göre, $x + y$ nin 0 ile π arasındaki değeri kaç radyandır ?

A) $\frac{\pi}{4}$ B) $\frac{\pi}{5}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{7}$ E) $\frac{\pi}{8}$

6. $\text{Sin}15 + \text{Cos}15 = ?$

A) $\frac{\sqrt{6}}{2}$ B) $\frac{\sqrt{6}}{4}$ C) $\frac{\sqrt{6}}{8}$
D) $\frac{\sqrt{2}}{4}$ E) $\frac{\sqrt{2}}{8}$

7. $\frac{\tan(180-\alpha) \cdot \text{Cos}\,(180-\alpha) \cdot \tan\,(90-\alpha)}{\text{Sin}\,(90+\alpha) \cdot \text{Cotg}\,(90-\alpha) \cdot \tan\,(90+\alpha)} = ?$

A) 1 B) -1 C) $\text{Cos}\,\alpha$ D) $\frac{1}{2}\,\text{Sin}2\alpha$ E) $\text{Sin}\,\alpha$

8. $\text{Cos}10^\circ \cdot \text{Cos}50^\circ \cdot \text{Cos}70^\circ = ?$

A) $\frac{\sqrt{3}}{4}$ B) $\frac{\sqrt{3}}{8}$ C) $\frac{\sqrt{3}}{16}$ D) $\frac{1}{16}$ E) $\frac{1}{8}$

9. $\hat{ABC}$ de $\tan A + \tan B + \tan C = m$ ise, $\tan A \cdot \tan B \cdot \tan C$ nin eşiti nedir ?

A) m^2 B) $2m$ C) $m^3 - 2n^2$ D) m E) $1 - m$

10. $\text{Sin}7x \cdot \text{Sin}3x - \text{Cos}7x \cdot \text{Cos}3x$ ifadesinin eşiti nedir ?

A) Cos4x B) $-$ Cos4x C) Cos10x
D) $-$ Cos10x E) Sin4x

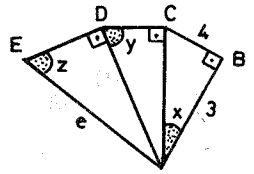
11. $\overset{\frown}{ABC}$ de $m\hat{B} = 60°$ ve $b = 6\sqrt{3}$ ise, $\overset{\frown}{ABC}$ nin çevrel çemberinin yarıçapı nedir ?

A) 2 B) 3 C) 6 D) $2\sqrt{3}$ E) $3\sqrt{3}$

12. $\text{Sin}75° \cdot \text{Cos}15° + \text{Cos}75° \cdot \text{Sin}15° = ?$

A) 0 B) 1 C) -1 D) $\sqrt{2}/5$ E) $\sqrt{3}/2$

13.



Şekilde,

$\text{Sin}\,x \cdot \text{Sin}\,y \cdot \text{Sin}\,z = \frac{1}{5}$

ise, $|AE| = e$ uzunluğu kaç birimdir ?

A) 13 B) 14 C) 16 D) 18 E) 20

1.C	2.C	3.C	4.B	5.A	6.A	7.B	8.A	9.D	10.D	11.C	12.B	13.E

TEST 07

1. $5^{\text{Cosx}} = \sqrt[4]{\frac{1}{25}}$ ise, x kaç olabilir ?

A) $\frac{\pi}{3}$ B) $\frac{2\pi}{3}$ C) $\frac{3\pi}{4}$ D) $\frac{5\pi}{6}$ E) $\frac{11\pi}{6}$

2. $\frac{\text{Sin}\,75 - \text{Sin}\,15}{\text{Cos}\,75 + \text{Cos}\,15}$ ifadesinin eşiti nedir ?

A) $\sqrt{3}$ B) $\frac{\sqrt{3}}{3}$ C) $\frac{\sqrt{2}}{2}$
D) $-\sqrt{3}$ E) $-\frac{\sqrt{3}}{3}$

3. $\text{Cos}\,100 + \text{Cos}\,20 + \text{Cos}\,220 = ?$

A) Cos 10 B) Sin 10 C) 0 D) 1 E) -1

4. $\frac{\text{SinA} + \text{Sin3A} - \text{Sin2A}}{\text{CosA} + \text{Cos3A} - \text{Cos2A}}$ nin eşiti nedir ?

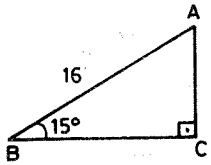
A) tgA B) CotgA C) Cotg2A D) Sin2A E) tg2A

5. $\frac{\text{Sin}\,3x}{\text{Sin}\,x} - \frac{\text{Cos}3x}{\text{Cos}x}$ eşiti kaçtır ?

A) 1 B) 2 C) -1 D) -2 E) 3

6. $\triangle ABC$ de $m\hat{A} = 60°$ ve $a = \sqrt{3}(b - c)$ dir. $c = 4$ ise, b kaç birimdir ?

A) 2 B) 3 C) 4 D) 6 E) 8

7. 

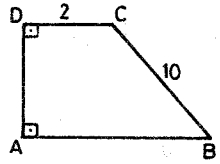
$A(\triangle ABC)$ kaç birim karedir ?

A) 32 B) 16 C) 14
D) 12 E) 10

8. $\cos\frac{11\pi}{12} \cdot \cos\frac{5\pi}{12}$ kaçtır ?

A) $-\frac{1}{4}$ B) $\frac{\sqrt{3}}{4}$ C) $\frac{1}{2}$

D) $-\frac{1}{2}$ E) $\frac{1}{4}$

9. 

Şekilde $\sin B = \frac{4}{5}$ ise,
A (ABCD) kaç birim karedir ?

A) 30 B) 36 C) 38
D) 40 E) 45

10. $0 < \alpha < \frac{\pi}{2}$; $\cos 2\alpha = \frac{3}{4}$ ise, $\sin\alpha$ kaçtır ?

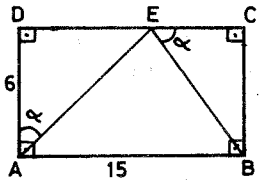
A) $\frac{\sqrt{2}}{4}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{2}}{3}$ D) $\frac{\sqrt{2}}{5}$ E) $\frac{1}{2}$

11. $\frac{\sin x + \cos x}{\sin x - 2\cos x} = \frac{3}{2}$ ise, tg x kaçtır ?

A) 8 B) 7 C) 6 D) 5 E) 4

12. $\tan\alpha = \frac{1}{2}$ ve $\tan\beta = \frac{1}{3}$ ise, $\alpha + \beta$ kaç derece olabilir ?

A) 30° B) 45° C) 60° D) 75° E) 90°

13. 

Şekile göre, tan α nın değerlerinden biri nedir ?

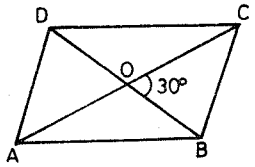
A) 1 B) 1/2
C) 1/3 D) 1/4
E) 1/5

1-B 2-B 3-C 4-E 5-B 6-E 7-A
8-A 9-D 10-A 11-A 12-B 13-B

TEST 08

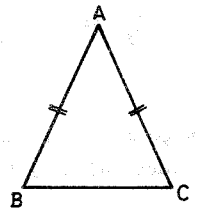
1. $\sin 10^\circ = m$ ise, $\cos 140^\circ + \cos 20^\circ$ kaçtır ?

A) m B) 2m C) m/4 D) m^2 E) $m^2/2$

2. 

ABCD dörtgeninde $|AC| = 10$, $|BD| = 6$ ise, A (ABCD) kaç birim karedir ?

A) 30 B) 25 C) 24
D) 20 E) 15

3. 

ABC üçgeninde $|AB| = |AC|$ dir. $\tan A = \frac{5}{12}$ ise, tanB nin değeri nedir ?

A) 5 B) 1/5 C) 4
D) 1/4 E) 12/13

4. $a = \frac{\pi}{20}$ ise, $\operatorname{tg}9a + \operatorname{tg} a - \operatorname{tg} 7a - \operatorname{tg} 3a$ nın değeri nedir?

A) 2 B) 3 C) 4 D) – 4 E) – 3

5. $\cos 4^\circ = m$ ise, $\sin 82^\circ$ kaçtır ?

A) m B) m + 1 C) $2m^2 - 1$ D) $1 - 2m^2$ E) $\frac{m^2+1}{2}$

6. $\overset{\frown}{ABC}$ de $a = 10$, $c = 15$ ve $\operatorname{Cotg} B = \frac{\sqrt{5}}{2}$ ise, A ($\overset{\frown}{ABC}$) kaç birim karedir ?

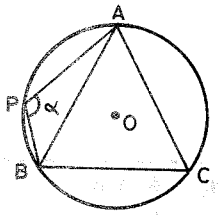
A) 40 B) 50 C) 55 D) 60 E) 75

7. $16 \cdot \sin\frac{\pi}{6} \cdot \sin\frac{5\pi}{6} \cdot \sin\frac{7\pi}{6} \cdot \sin\frac{11\pi}{6}$ kaçtır ?

A) 1 B) – 1 C) 2 D) – 2 E) 4

8. $\tan 1 \cdot \tan 2 \cdot \tan 3 \cdot \tan 4 \ldots\ldots \tan 87 \cdot \tan 88 \cdot \tan 89$ kaçtır?

A) 1 B) ∞ C) $-\infty$ D) – 1 E) 0

9. 

ABC eşkenar üçgeninin çevrel çemberi üzerinde bir P noktası alınmıştır. $\operatorname{tg}\alpha$ kaçtır ?

A) $-\sqrt{3}$ B) $\sqrt{3}$ C) $\frac{\sqrt{2}}{2}$
D) $-\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{3}}{3}$

10. $\alpha = 10^\circ$ ise, $\frac{\text{Cos8}\,\alpha + \text{Cos4}\,\alpha}{\text{Sin8}\,\alpha + \text{Sin}\,4\,\alpha}$ nın değeri kaçtır ?

A) 1 B) $-\sqrt{3}$ C) $\sqrt{3}$ D) $\sqrt{3}/3$ E) $-\sqrt{3}/3$

11. $\frac{1}{\text{Sin}\,10^\circ} - \frac{\sqrt{3}}{\text{Cos}\,10^\circ}$ ifadesinin eşiti kaçtır ?

A) 1/4 B) 1/2 C) 1 D) 2 E) 4

12. $\widehat{ABC}$ de $a = 2 \cdot b \cdot \text{Cos}C$ dır. $m\hat{A} = 80^\circ$ ise, $m\hat{B}$ kaç derecedir ?

A) 30° B) 40° C) 50° D) 60° E) 80°

13. $\frac{\text{Cos}\,x + \text{Cos}\,6x + \text{Cos}\,11x}{\text{Sin}\,x + \text{Sin}\,6x + \text{Sin}\,11x} = ?$

A) Cot 6x B) Sin 18 x C) tan 6x
D) 1 E) 0

1-A 2-E 3-A 4-C 5-C 6-B 7-A
8-A 9-A 10-D 11-E 12-C 13-A

TEST 09

1. $\frac{1}{\text{Sin}\,10} - 4\,\text{Sin}\,70$ kaçtır ?

A) 0 B) 1 C) – 1 D) 2 E) – 2

2. $\frac{\text{Cos}\,12}{\text{Cos}\,4} - \frac{\text{Sin}\,12}{\text{Sin}\,4}$ kaçtır ?

A) 2 B) 1 C) – 1 D) – 2 E) – 4

3. $\pi < x < 2\,\pi$ iken $2\,\text{Sin}^2 x + 5\,\text{Cos}x + 1 = 0$ denkleminin kökü aşağıdakilerden hangisidir ?

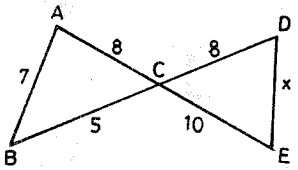
A) $\frac{7\pi}{6}$ B) $\frac{5\pi}{3}$ C) $\frac{3\pi}{2}$ D) $\frac{11\pi}{6}$ E) $\frac{4\pi}{3}$

4. $\alpha = \frac{\pi}{13}$ ise, $\frac{\text{Cos4}\,\alpha + \text{Cos2}\alpha}{\text{Cos}\,\alpha \cdot \text{Cos10}\alpha}$ ifadesinin eşiti nedir ?

A) – 2 B) – 1 C) 1/2 D) 1 E) 2

5. $\text{Sin}^4 75 + \text{Cos}^4 75$ ifadesinin eşiti kaçtır ?

A) 3/4 B) 3/8 C) 5/8 D) 7/8 E) 1/4

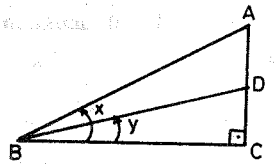
6. 

Şekilde x kaçtır ?

A) 5 B) 6
C) $\sqrt{21}$ D) $2\sqrt{21}$
E) $\sqrt{23}$

7. Sin (135 + x) + Sin (135 – x) ifadesinin en sade şekli nedir ?

A) $\sqrt{2}$. Sin x B) $\sqrt{3}$. Cos x C) – 1
D) $\sqrt{2}$. Cos x E) $\sqrt{3}$. Sin x

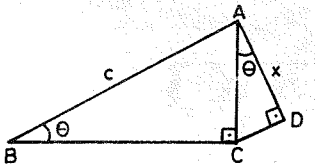
8. 

Şekilde $m\hat{C} = 90°$, $m\widehat{ABC} = x$, $m\widehat{DBC} = y$ iken, $\frac{|AB| \cdot \text{Sin}(x-y)}{\text{Cos } y}$ ifadesinin eşiti aşağıdakilerden hangisidir?

A) |BD| B) |AB| C) |AC| D) |CD| E) |AD|

9. $A = \frac{5 + \text{Cotg}^4(\pi/6) - \tan^2(\pi/4)}{\text{Cotg}(\pi/4) - 4\,\text{Cos}^2(\pi/3) - 8\,\text{Sin}^3(\pi/6)}$ ise, A nın sayı değeri kaçtır ?

A) – 1 B) – 7 C) – 8 D) – 9 E) – 13

10. 

Şekildeki verilere göre, |AD| = x nedir ?

A) c . Sin θ B) c . tan θ C) c . Cos θ
D) c . Sin 2 θ E) $\frac{1}{2}$. c . Sin 2 θ

11. $\frac{\text{Cos}115 \cdot \text{Sin}305 + \text{Sin}35 \cdot \text{Cos}25}{\text{Cos}140 \cdot \text{Sin}230 - \text{Cos}50 \cdot \text{Sin}220}$ ifadesinin eşiti aşağıdakilerden hangisidir ?

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $\sqrt{3}/2$ D) 1 E) – 1

12. Arc tan $\frac{1}{3}$ + Arc tan $\frac{1}{2}$ ifadesinin eşiti aşağıdakilerden hangisidir ?

A) π B) $\pi/2$ C) $\pi/3$ D) $\pi/4$ E) $\pi/6$

1-D 2-D 3-E 4-A 5-D 6-D 7-D 8-E 9-E 10-E 11-C 12-D

TEST 10

1. $\tan\frac{A}{2} + \text{Cotg}\frac{A}{2}$ nin eşiti nedir ?

A) 2 SinA B) 2CosA C) 2 SecA
D) 2 tan A E) 2CosecA

2. $\text{Cos}\frac{2\pi}{5} \cdot \text{Cos}\frac{\pi}{5}$ in eşiti nedir ?

A) 1/8 B) 1/6 C) 1/4 D) 1/2 E) 1

3. $\text{Sin}15^\circ \cdot \text{Cos}15^\circ$ in eşiti nedir ?

A) 1/8 B) 1/6 C) 1/4 D) 1/2 E) 1

4. $x = \frac{\pi}{10}$ ise,

$2 \cdot \text{Cos}\,\pi \cdot \text{Sin}\,(\pi - x) \cdot \text{Sin}\,(\frac{3\pi}{2} + x) = ?$

A) $\text{Sin}\frac{\pi}{5}$ B) $\text{Sin}\frac{2\pi}{5}$ C) $\text{Sin}\frac{4\pi}{5}$
D) $\text{Cos}\frac{\pi}{5}$ E) $\text{Cos}\frac{2\pi}{5}$

5. $f(x) = \frac{1}{7}(\text{Cos}^2 x - \text{Sin}^2 x)$ fonksiyonunun periyodu nedir ?

A) π B) $\pi/2$ C) $\pi/4$ D) 2π E) $3\pi/2$

6. $\text{Cos}x - \text{Sin}x = \frac{1}{\sqrt{2}}$ ise, x in en küçük pozitif değeri kaç derecedir ?

A) 5° B) 12° C) 15° D) 18° E) 30°

7. $x = \frac{\pi}{20}$ ise $\frac{\text{Cos}4x - \text{Cos}8x}{\text{Cos}8x \cdot \text{Cos}4x} = ?$

A) $-1/2$ B) -1 C) -2 D) 2 E) 1

8. $\hat{ABC}$ de $b = \sqrt{3} + 1$, $c = \sqrt{3} - 1$ ve $m\hat{A} = 60^\circ$ ise, a kaç birimdir ?

A) $\sqrt{5}$ B) $\sqrt{6}$ C) $\sqrt{7}$ D) $2\sqrt{2}$ E) 3

9. $\text{Cos}20 \cdot \text{Cos}40 \cdot \text{Cos}60 \cdot \text{Cos}80 = ?$

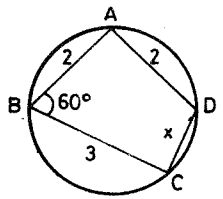
A) 1/2 B) 1/4 C) 1/8 D) 1/16 E) 1/32

10. Sin20 . Sin40 . Sin60 . Sin80 = ?

A) 1/16 B) 1/32 C) 1/8 D) 3/16 E) $\sqrt{3}/16$

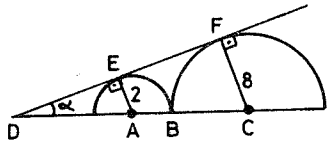
11. $\frac{Sina + Sin3a + Sin5a}{Cos5a + Cos3a + Cosa} = ?$

A) tga B) tg2a C) Cotga D) Cotg2a E) tg3a

12. 

Şekilde ABCD kirişler dörtgeni ise, |CD| = x kaç birimdir ?

A) 1 B) 2 C) 3
D) 5/2 E) 7/2

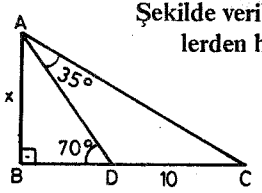
13. 

Şekilde A ve C noktaları çemberlerin merkezi ve $m(\widehat{EDA}) = \alpha$ olduğuna göre, tan α nın değeri nedir ?

A) 5/6 B) 4/5 C) 3/4 D) 2/3 E) 1/2

1-E 2-C 3-C 4-A 5-A 6-C 7-D
8-B 9-D 10-D 11-E 12-A 13-C

TEST 11

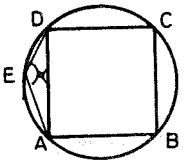
1. 

Şekilde verilenlere göre, |AB| = x aşağıdakilerden hangisidir ?

A) 10 Sin 70 B) 10 Cos 70
C) 5 Cos 20 D) 5 Cos 70
E) 2 Sin 10

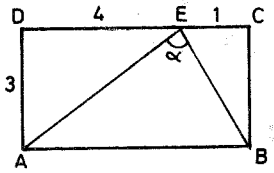
2. $A = \{ x \in R : Cos\, x \leq \frac{1}{2} \}$ kümesine birim çember üzerinde karşılık gelen noktaların oluşturduğu yayın uzunluğu kaç birimdir ?

A) $\pi/6$ B) $\pi/4$ C) $\pi/3$ D) $2\pi/3$ E) $4\pi/3$

3. 

Şekilde ABCD bir karedir. Cotg α aşağıdakilerden hangisidir ?

A) $\sqrt{3}$ B) $-\sqrt{3}$ C) 1
D) -1 E) $\frac{\sqrt{3}}{3}$

4. 

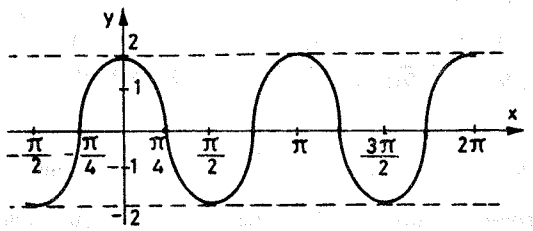
ABCD dikdörtgeninde verilenlere göre, tan α nın eşiti nedir ?

A) 1 B) 2 C) 3
D) 4 E) 5

5. $\frac{\text{Cos } 51^\circ}{\text{Cos } 17^\circ} - \frac{\text{Sin } 51^\circ}{\text{Sin } 17^\circ}$ ifadesinin eşiti aşağıdakilerden hangisidir ?

A) -4 B) -2 C) -1 D) 2 E) 4

6.

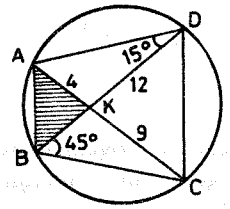


Şekildeki grafiğin denklemi aşağıdakilerden hangisidir ?

A) $y = \text{Cos } x$ B) $y = \text{Cos } 2x$ C) $y = 2\,\text{Cos}x$

D) $y = 2\,\text{Cos } 2x$ E) $y = 2\,\text{Cos } \frac{x}{2}$

7.



Şekilde ABCD kirişler dörtgenidir. Verilenlere göre, $A(\overset{\frown}{AKB})$ kaç birim karedir ?

A) 3 B) $2\sqrt{3}$ C) $3\sqrt{3}$
D) $4\sqrt{3}$ E) 6

8. $\overset{\frown}{ABC}$ de, $m\hat{A} = 45^\circ$, $|AB| = 3\sqrt{2}$ ve $|AC| = 7$ ise, $\overset{\frown}{ABC}$ nin çevrel çemberinin yarıçapı kaç birimdir ?

A) $2\sqrt{5}$ B) $\frac{5\sqrt{2}}{2}$ C) $6\sqrt{5}$ D) 5 E) $3\sqrt{2}$

9. $\pi < \alpha < \frac{3\pi}{2}$, $\text{tg } \alpha = 3$ ise,

$\text{Sec } \alpha\,[\,\text{Sin}^3\,\alpha\,\text{Cos}\,\alpha + \text{Cos}^3\,\alpha\,\text{Sin}\,\alpha\,]$ nin eşiti nedir ?

A) $\frac{3\sqrt{10}}{10}$ B) $\frac{-3\sqrt{10}}{10}$ C) $\frac{\sqrt{10}}{10}$

D) $\frac{-\sqrt{10}}{10}$ E) $\frac{-1}{3}$

10. $\overset{\frown}{ABC}$ de, $b = \sqrt{8}$, $c = \sqrt{12}$ ve $m\hat{C} = 60^\circ$ ise, $m(\hat{A})$ aşağıdakilerden hangisidir ?

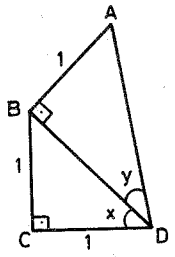
A) 15° B) 30° C) 45° D) 60° E) 75°

1-A	2-E	3-D	4-C	5-B	6-D	7-C	8-B	9-B	10-E

TEST 12

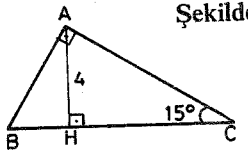
1. $\frac{\text{Sin}20^\circ}{\text{Sin}200^\circ} + \frac{\text{tg}\,220^\circ}{\text{Cotg}\,50^\circ} + \frac{\text{Cos}340^\circ}{\text{Sin}\,110^\circ}$ ifadesinin en sade şekli nedir ?

A) -2 B) -1 C) 0 D) 1 E) $+2$

2. 

Şekilde verilenlere göre, Sin $(x + y)$ kaçtır ?

A) $\frac{2\sqrt{3}+\sqrt{6}}{6}$ B) $\frac{2\sqrt{3}+1}{6}$ C) $\frac{2+\sqrt{3}}{2}$ D) $\frac{\sqrt{2}+2}{3}$ E) $\frac{2-\sqrt{2}}{3}$

3. 

Şekilde |BC| kaç birimdir ?

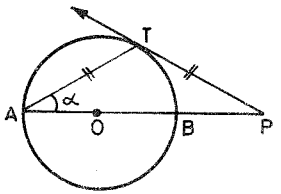
A) 6 B) 8 C) 10 D) 12 E) 16

4. $\frac{\text{Sin}\,15^\circ}{\text{Sin}\,5^\circ} - \frac{\text{Cos}15^\circ}{\text{Cos}\,5^\circ}$ ifadesinin eşiti nedir ?

A) 1 B) 2 C) -2 D) -1 E) 0

5. $\frac{\text{Sin}\,15^\circ}{\sqrt{6}-\sqrt{2}}$ kaçtır ?

A) 2 B) 4 C) 1/2 D) 1/4 E) $-1/4$

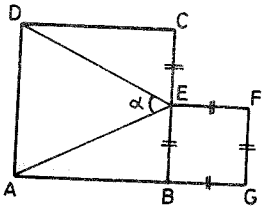
6. 

Şekilde [PT teğet ve O çemberin merkezidir. |AT| = |TP| ise, Sin α nedir ?

A) 1/2 B) $\sqrt{3}/2$ C) $\sqrt{2}/2$ D) $-1/2$ E) $-\sqrt{3}/2$

7. $a \cdot b = 1$ ve $a^{\text{Sin}x} = b^{\text{Cos}x}$ ise, x aşağıdakilerden hangisi olabilir ?

A) π B) $\pi/2$ C) $\pi/3$ D) $\pi/6$ E) $3\pi/4$

8. 

ABCD ve BGFE karedir. |BE| = |EC| ise, tan α kaçtır ?

A) 1 B) 1/3 C) 2/3 D) 3/4 E) 4/3

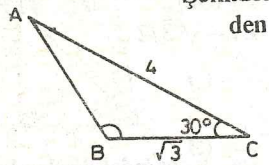
9. $\frac{1 + \cos 2x}{1 - \cos 2x}$ ifadesinin sadeleşmiş hali nedir ?

A) 1 B) Sin2x C) $\cot g^2 x$ D) $\tan^2 x$ E) $\sin^2 x$

10. $\sin 18 = a$ ise, $\sin^2 24 - \sin^2 6$ kaçtır ?

A) a B) 2a C) $\sqrt{a}$ D) a/2 E) a^2

11. Şekildeki verilere göre, SinB aşağıdakilerden hangisidir ?



A) $\frac{2\sqrt{7}}{7}$ B) $\sqrt{\frac{11}{15}}$ C) 3/8 D) 4/7 E) $\sqrt{7}/7$

12. $\triangle ABC$ de $a = 6$, $c = 4$ ve $m\hat{A} = 2 \cdot m\hat{C}$ ise, CosC nin değeri nedir ?

A) 2/3 B) 3/4 C) 3/5 D) 4/5 E) 1/2

1-D 2-A 3-E 4-B 5-D 6-A 7-E 8-E 9-C 10-D 11-A 12-B

TEST 13

1. $\triangle ABC$ de $(\cos B + \cos A \cdot \cos C - \sin A \cdot \sin C)$ ifadesinin değeri nedir ?

A) 1 B) -1 C) 0 D) 1/2 E) $1/\sqrt{3}$

2. $6 - 2 \sin x \cdot \cos x$ ifadesi x in hangi değeri için minimum olur ?

A) $\frac{\pi}{4}$ B) $\frac{\pi}{2}$ C) $\frac{5\pi}{3}$ D) $\frac{3\pi}{2}$ E) $\frac{\pi}{4}$ ve $\frac{5\pi}{4}$

3. $x \in [0, 2\pi)$ iken $\cos 2x - \cos x + 1 = 0$ denkleminin çözüm kümesi nedir ?

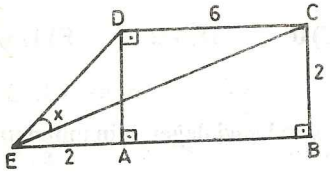
A) $\{\frac{\pi}{4}, \frac{\pi}{6}\}$ B) $\{\frac{\pi}{2}, \frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{6}\}$ C) $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$ D) $\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}\}$ E) ϕ

4. $\tan x + \cot g x = m$ ise, $\tan^2 x + \cot g^2 x$ kaçtır ?

A) m^2 B) 2m C) $m^2 + 2$ D) $m^2 - 2$ E) m

5. $\sin^2 390 + \sin^2 480$ ifadesinin değeri nedir ?

A) 1 B) 0 C) $\sqrt{2}/2$ D) $\sqrt{3}/2$ E) 2

6. 

Şekilde verilere göre, tan x kaçtır ?

A) 3/5 B) 3/4 C) 4/3 D) 5/3 E) 5/4

7. $(\cos 70^\circ \cdot \cos 290^\circ + \cos 20^\circ \cdot \cos 380^\circ)$ kaçtır ?

A) -2 B) -1 C) 0 D) 1 E) 2

8. $\dfrac{\cos^3 x + \sin^3 x + \sin x \cos x (\sin x + \cos x)}{\sin x + \cos x}$ ifadesinin en sade şekli nedir ?

A) 1 B) 2 C) Sinx + Cosx D) tgx E) Cotg

9. $\sin 2x = 0{,}6$ ise, tanx – Cotgx kaçtır ?

A) $-3/4$ B) $-3/2$ D) $-8/3$ D) -2 E) -1

10. $\sin 195^\circ \cdot \cos 165^\circ$ kaçtır ?

A) 1/2 B) $-1/2$ C) $-1/4$ D) 1/4 E) $\sqrt{2}/2$

11. $\sin 10^\circ = a$ ise, $\sin 470^\circ$ kaçtır ?

A) $2a^2 - 1$ B) $1 - 2a^2$ C) $1 + a^2$
D) $1 - a^2$ E) a^2

12. $a, b \in [0, \dfrac{\pi}{2})$ iken,

$\sin a = \dfrac{3}{5}$ ve $\sin b = \dfrac{5}{13}$ ise $\sin(a + b)$ kaçtır ?

A) 56/65 B) 53/65 C) 62/65 D) 58/65 E) 51/5

13. Sin 95°, Cos 190°, tan 210° nin işaretleri aşağıdakilerden hangisinde doğru olarak verilmiştir ?

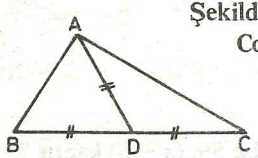
A) +, −, − B) −, −, +
C) −, +, + D) +, +, −
E) +, −, +

1-C 2-E 3-D 4-D 5-A 6-A 7-D
8-A 9-D 10-D 11-B 12-A 13-E

TEST 14

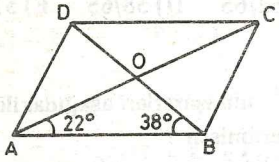
1. $n \in N$ ise, $\text{Sin}\,[\,(-1)^n \,.\, (\frac{\pi}{2} - n\,\pi)\,]$ ifadesinin en sade yazımı nedir ?

A) $(-1)^n$ B) $(-1)^{n-1}$ C) 0 D) -1 E) 1

2. 

Şekilde $|AD| = |BD| = |DC|$ dir. $tgB = 3$ ise, CotgC nedir ?

A) 3 B) 1/3 C) 6
D) 1/6 E) 1/2

3. 

ABCD paralelkenardır. $|AC| = 8$, $|BD| = 6$ ise, A (ABCD) kaç birim karedir ?

A) $12\sqrt{3}$ B) 12 C) 24
D) $24\sqrt{3}$ E) 48

4. $\overset{\triangle}{ABC}$ de $\text{SinA} = 2\,\text{SinB}\,.\,\text{CosC}$ ve $b = 7$ ise, c kaç birimdir ?

A) 7/2 B) 7 C) 8 D) 9 E) 14

5. $\overset{\triangle}{ABC}$ de $\dfrac{\text{Sin B}}{\text{Cos C}} = \text{SinA} + \text{CosA}\,.\,\text{CotgB}$ ise, SinA kaçtır ?

A) 1 B) 0 C) $\sqrt{2}/2$ D) -1 E) $-\sqrt{2}/2$

6. $\overset{\triangle}{ABC}$ de $a = \dfrac{\sqrt{3}}{2}$, $b = \dfrac{\sqrt{2}}{2}$, $c = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ ise, $m\hat{B}$ kaç derecedir ?

A) 30 B) 45 C) 60 D) 120 E) 150

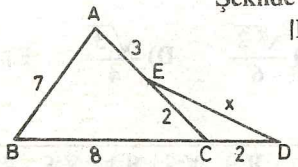
7. $x \in [\,\frac{\pi}{2}\,,\,\frac{3\,\pi}{2}\,]$ iken $4\,\text{Sin}^2 x - 3 = 0$ denkleminin çözüm kümesi nedir ?

A) $\{\frac{2\pi}{3}\}$ B) $\{\frac{4\,\pi}{3}\}$ C) $\{\frac{5\,\pi}{3}\}$

D) $\{\frac{2\pi}{3}\,,\,\frac{4\,\pi}{3}\}$ E) $\{\frac{\pi}{3}\,,\,\frac{2\,\pi}{3}\,,\,\frac{4\,\pi}{3}\,,\,\frac{5\,\pi}{3}\}$

8. $\overset{\triangle}{ABC}$ de $c^2 - ab = a^2 + b^2$ ise, $m\hat{C}$ kaç derecedir ?

A) 30 B) 60 C) 120 D) 135 E) 150

9. 

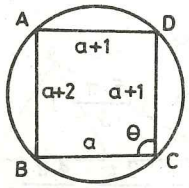
Şekilde B, C, D doğrusal ise verilenlere göre $|ED| = x$ kaç birimdir ?

A) $\sqrt{3}$ B) $2\sqrt{3}$
C) $3\sqrt{3}$ D) $2\sqrt{5}$
E) $2\sqrt{7}$

10. $Cos^2x + Cos2x = Sin^2x + Sin2x$ denklemini sağlayan en küçük dar açının tanjantı kaçtır ?

A) $\frac{\sqrt{5}-1}{2}$ B) $2+\sqrt{5}$ C) $1+\sqrt{5}$

D) $\frac{\sqrt{5}}{2}$ E) $\frac{\sqrt{5}+1}{2}$

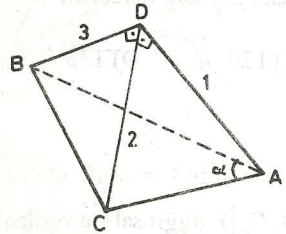
11. 

Şekilde Cos θ kaçtır ?

A) $\frac{1}{a+1}$ B) $\frac{a}{a+1}$ C) $\frac{a}{a+2}$

D) $-\frac{1}{a+2}$ E) $-\frac{1}{a+1}$

12. Aşağıdaki şekilde verilen DABC dörtyüzlüsünün D köşesinden geçen ayrıtları birbirine diktir.



$|DB| = 3$ cm,
$|DC| = 2$ cm,
$|DA| = 1$ cm

Cos α nın değeri nedir ?

A) $\frac{\sqrt{2}}{10}$ B) $\frac{\sqrt{2}}{8}$ C) $\frac{\sqrt{2}}{6}$ D) $\frac{\sqrt{3}}{4}$ E) $\frac{\sqrt{2}}{4}$

1-E 2-A 3-A 4-B 5-A 6-B 7-D 8-C 9-B 10-A 11-E 12-A

TEST 15

1. $\widehat{ABC}$ de $tgB = CotgC$ ise, $\frac{SinB + CosC}{SinC + CosB}$ ifadesinin en sade şekli nedir ?

A) SinB B) tgA C) tgB D) CosB E) SinB

2. $tan25 = a$ ise, $\frac{tan205 + tan115}{tan245 + tan335}$ kaçtır ?

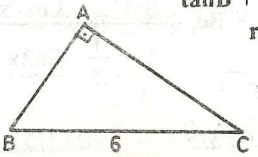
A) $\frac{1+a^2}{1-a^2}$ B) $\frac{a^2-1}{a^2}$ C) $\frac{a^2+1}{a^2}$

D) $\frac{a+1}{a^2+1}$ E) -1

3. $\widehat{ABC}$ de $CosB = \frac{3}{5}$ ve $b = 20$ ise, bu üçgenin çevrel çemberinin yarıçapı nedir ?

A) 10 B) 25 C) 25/2 D) 15 E) 25/8

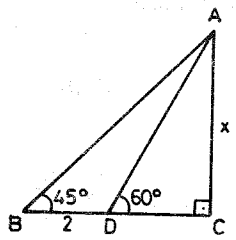
4. $tanB + tanC = m$ ise, $A(\widehat{ABC})$ kaç birim karedir ?



A) 9/m B) 10/m C) 18/m
D) 36/m E) 72/m

5. $\text{Sin}114^\circ = a - 1$ ise, $\text{Cos}12^\circ$ kaçtır ?

A) $\frac{a}{2}$ B) $\sqrt{\frac{a}{2}}$ C) $\frac{\sqrt{a}}{2}$ D) $\frac{a-1}{2}$ E) a

6. 

Şekilde x kaç birimdir ?

A) $\frac{\sqrt{3}}{2}$ B) $\sqrt{3}$
C) 2 D) $3+\sqrt{3}$
E) 6

7. Aşağıdakilerden hangisi $\text{Sin}(\frac{\pi}{2} - a)$ ya özdeş değildir ?

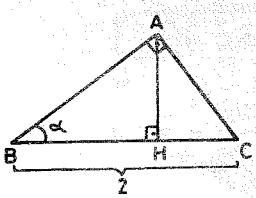
A) $\text{Sin}(\frac{\pi}{2} + a)$ B) $\text{Sin}(-a)$ C) $\text{Cos}(-a)$
D) Cos a E) $\text{Cos}(2\pi - a)$

8. $0 < \alpha < \frac{\pi}{2}$ ve $\text{Cos}\,\alpha = \frac{\sqrt{3}}{3}$ ise, Cos2 α aşağıdakilerden hangisidir ?

A) 1/2 B) 1/3 C) – 1/2 D) – 1/3 E) – 3/4

9. $0 < x < \frac{\pi}{2}$ ve $\tan x = \frac{4}{3}$ ise, $\frac{\text{Sin}^3 x - \text{Cos}^3 x}{1 + \frac{1}{2}\text{Sin}2x}$ ifadesinin eşiti nedir ?

A) 1/5 B) 2/5 C) 3/5 D) 4/5 E) 1

10. 

Şekilde |BC| = 2 ise, |AH| nın değeri nedir ?

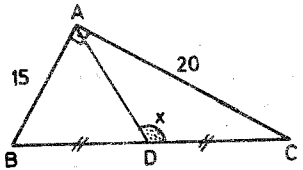
A) Sin 2 α B) 1 – Sin 2 α
C) 1 + Sin2 α D) 2 – Sin 2 α
E) 2 + Sin2 α

11. $10\,a = \frac{\pi}{2}$ ise, $\frac{\text{Cos}4a - \text{Cos}8a}{\text{Cos}4a \,.\, \text{Cos}8a}$ ifadesinin değeri nedir ?

A) – 1/2 B) – 1 C) – 2 D) 1 E) 2

12. $\text{Cos}^2 \frac{\pi}{8} + \text{Sin}^2 \frac{3\pi}{8} = \frac{a}{2}$ ise, a kaçtır ?

A) $\frac{1}{2}$ B) $\frac{\sqrt{2}}{2}$ C) $2+\sqrt{2}$ D) 1 E) $\sqrt{2}-1$

13. 

Şekildeki dik üçgende [AD] kenarortay ise, Cos x değeri nedir ?

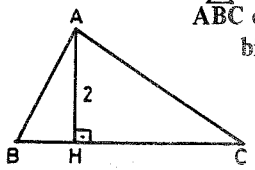
A) 12/25 B) 11/25 C) 8/25 D) 7/25 E) 6/25

1-C	2-E	3-C	4-C	5-B	6-D	7-B	8-D	9-A	10-A	11-E	12-C	13-D

TEST 16

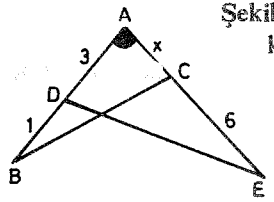
1. $\widehat{ABC}$ de $b^2 = a^2 + c^2 - \sqrt{2}\,ac$ ve $b = 4\sqrt{2}$ ise, bu üçgenin çevrel çemberinin yarıçapı nedir ?

A) 4 B) 3 C) 2 D) 1 E) $\sqrt{2}$

2. 

$\widehat{ABC}$ de CotgB + CotgC = 6 ise, A($\widehat{ABC}$) kaç birim karedir ?

A) 6 B) 8 C) 10
D) 12 E) 24

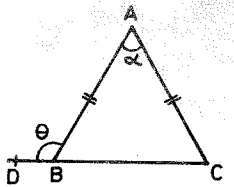
3. 

Şekilde A ($\widehat{ADE}$) = 3 . A ($\widehat{ABC}$) ise, x kaç birimdir ?

A) 2 B) 4 C) 5
D) 6 E) 7

4. $\cos^2 \frac{3\pi}{8} - \sin^2 \frac{3\pi}{8}$ ifadesinin eşiti nedir ?

A) $\frac{\sqrt{2}}{2}$ B) $-\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $-\frac{\sqrt{3}}{2}$ E) 1

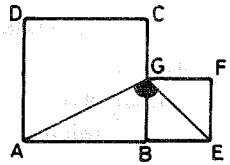
5. 

Şekilde Cotg $\alpha = \frac{12}{5}$ ve |AB| = |AC| ise, tan θ kaçtır ?

A) 13 B) – 13 C) 5
D) – 5 E) 12

6. $\frac{\sin^2 15 + \sin^2 75}{\sin 75 \,.\, \sin 15}$ ifadesinin değeri kaçtır ?

A) 4 B) 1/4 C) 2 D) 1/2 E) $\sqrt{2}/2$

7. 

Şekilde ABCD ve BEFG karedir. |GC| = |GB| ise, Sin ($\widehat{AGE}$) nin değeri nedir ?

A) $\frac{3\sqrt{2}}{4}$ B) $\frac{5\sqrt{3}}{3}$ C) $\frac{2\sqrt{5}}{5}$

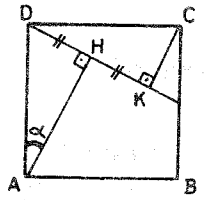
D) $\frac{2\sqrt{7}}{7}$ E) $\frac{3\sqrt{10}}{10}$

8. $\widehat{ABC}$ de $m\hat{A} = 60°$ ve $a = \sqrt{3}(b - c)$ ise, B ve C açılarının ölçüleri sırasıyla nedir ?

A) 70° ; 50° B) 90° ; 30° C) 60° ; 60°
D) 80° ; 40° E) 50° ; 70°

9. $\text{Sin}2x = \text{Cos}35^\circ$ denkleminin $[0^\circ, 90^\circ]$ aralığındaki kökü kaç derecedir ?

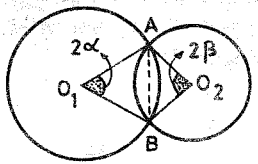
A) 70° B) 60° C) $37{,}5^\circ$ D) $27{,}5^\circ$ E) $17{,}5^\circ$

10. 

Şekilde ABCD bir kare ve $|DH| = |HK|$ dır. $\text{tg}\,\alpha$ kaçtır ?

A) $\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{3}{4}$

D) $\frac{2}{3}$ E) $\frac{1}{2}$

11. 

Şekildeki A ve B noktalarında kesişen çemberlerin yarıçaplarının oranı aşağıdakilerden hangisidir ?

A) $\frac{\text{Sin}\,\beta}{\text{Sin}\,\alpha}$ B) $\frac{\text{Cos}\,\beta}{\text{Cos}\,\alpha}$ C) 1

D) $\frac{\text{Cot}\,\alpha}{\text{Cot}\,\beta}$ E) $\frac{\text{Cos}\,\alpha}{\text{Cos}\,\beta}$

1-A	2-D	3-A	4-B	5-D	6-A
7-E	8-B	9-D	10-E	11-A	

10. BÖLÜM

EK ÇALIŞMA SORULARI
(A)

1. Aşağıdakileri ispatlayınız.

a) $\text{Sec}(\frac{\pi}{4}+\alpha)\,.\,\text{Sec}(\frac{\pi}{4}-\alpha) = 2\,\text{Sec}\,2\alpha$

b) $\frac{\text{Sin}\,(2\alpha+\beta)}{\text{Sin}\,\alpha} - 2\,\text{Cos}\,(\alpha+\beta) = \frac{\text{Sin}\,\beta}{\text{Sin}\,\alpha}$

c) $\frac{\text{Cos}\,\alpha + \text{Sin}\,\alpha}{\text{Cos}\,\alpha - \text{Sin}\,\alpha} = \tan(\frac{\pi}{4}+\alpha)$

d) $\frac{\text{Cos}\,\alpha + \text{Sin}\,\alpha}{\text{Cos}\,\alpha - \text{Sin}\,\alpha} = \tan 2\alpha + \text{Sec}\,2\alpha$

e) $\frac{\text{Cos}\,2\alpha}{\text{Cotg}^2\,\alpha - \tan^2\alpha} = \frac{1}{4}\,\text{Sin}^2\,2\alpha$

f) $\frac{1+\text{Sin}\,2\alpha}{\text{Cos}\,2\alpha} = \tan(\frac{\pi}{4}+\alpha)$

g) $\frac{\text{Sin}\,\alpha - \text{Sin}\,3\alpha + \text{Sin}\,5\alpha}{\text{Cos}\,\alpha - \text{Cos}\,3\alpha + \text{Cos}5\alpha} = \tan 3\alpha$

h) $A+B+C=\pi$ ise,

$\text{CotgA}\,.\,\text{CotgB} + \text{CotgA}\,.\,\text{CotgC} + \text{CotgB}\,.\,\text{CotgC} = 1$

2. Aşağıdaki denklemleri çözünüz.

a) $1 - \text{Sin}\,5x = (\text{Cos}\,\frac{3x}{2} - \text{Sin}\,\frac{3x}{2})^2$

b) $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$

c) $\sin(x + 30) + \cos(x + 60) = 1 + \cos 2x$

d) $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$

e) $\cos 4x = -2\cos^2 x$

f) $\sin 3x = \cos 2x$

g) $\cos^2 x + 3\sin^2 x + 2\sqrt{3}\sin x \cdot \cos x = 1$

h) $\sin x + \sqrt{3} \cdot \cos x = 1$

i) $\sin x + \cos x = 1$

j) $\sin 3x + \cos 3x = \sqrt{2}$

k) $5\cos 2x = 4\sin x$

l) $\cot g\, x + \dfrac{\sin x}{1 + \cos x} = 2$

m) $\sin(\pi - x) + \cot g\left(\dfrac{3\pi}{2} - x\right) = \sec(-x) - \cos(2\pi - x)$

n) $\tan x + \tan 2x = \tan 3x$

o) $\cos x + \sin x = \dfrac{\cos 2x}{1 - \sin 2x}$

p) $\tan(x + \alpha) + \tan(x - \alpha) = 2\cot g\, x$

3. Aşağıdaki denklem sistemlerini çözünüz.

a) $\left.\begin{array}{l} x + y = \alpha \\ \sin x \cdot \sin y = b \end{array}\right\}$

b) $\left.\begin{array}{l} x + y = \alpha \\ \tan x + \tan y = b \end{array}\right\}$

c) $\left.\begin{array}{l} x + y = \dfrac{\pi}{4} \\ \tan x + \tan y = 1 \end{array}\right\}$

d) $\left.\begin{array}{l} 2^{\sin x + \cos y} = 1 \\ 16^{\sin^2 x + \cos^2 y} = 4 \end{array}\right\}$

e) $\left.\begin{array}{l} \sin x = 2\sin y \\ \cos x = \dfrac{1}{2}\cos y \end{array}\right\}$

EK ÇALIŞMA SORULARI
(B)

1. Aşağıdakileri ispatlayınız.

a) $\dfrac{1 + \sin\alpha}{1 + \cos\alpha} \cdot \dfrac{1 + \sec\alpha}{1 + \operatorname{cosec}\alpha} = \tan\alpha$

b) $\dfrac{1 - 2\sin^2\alpha}{1 + \sin 2\alpha} = \dfrac{1 - \tan\alpha}{1 + \tan\alpha}$

c) $(1 + \tan\alpha \cdot \tan 2\alpha) \cdot \sin 2\alpha = \tan 2\alpha$

d) $\dfrac{\sin^2 4\alpha}{2\cos\alpha + \cos 3\alpha + \cos 5\alpha} = 2\sin\alpha \cdot \sin 2\alpha$

e) $\sin 2\alpha + \sin 4\alpha - \sin 6\alpha = 4\sin\alpha \cdot \sin 2\alpha \cdot \sin 3\alpha$

f) $\dfrac{\sin 2x - \sin 3x + \sin 4x}{\cos 2x - \cos 3x + \cos 4x} = \tan 3x$

g) $\dfrac{\sin(\pi - \alpha)}{\sin\alpha - \cos\alpha \tan\left(\dfrac{\alpha}{2}\right)} + \cos(\pi - \alpha) = 1$

h) $\frac{1-\text{Sin}2\alpha}{1+\text{Sin}2\alpha} = \text{Cotg}^2\left(\frac{\pi}{4}+\alpha\right)$

i) $\text{Sin}^6\alpha + \text{Cos}^6\alpha + 3\,\text{Sin}^2\alpha \,.\, \text{Cos}^2\alpha = 1$

j) $\text{Sin}^2\left(\frac{\pi}{8}+\alpha\right) - \text{Sin}^2\left(\frac{\pi}{8}-\alpha\right) = \frac{\text{Sin}\,2\alpha}{\sqrt{2}}$

k) $\frac{\text{Cotg}\,\alpha + \tan\alpha}{1+\tan 2\alpha \,.\, \tan\alpha} = 2\,\text{Cotg}\,2\alpha$

l) $\frac{1-\text{Cos}\,2\alpha + \text{Sin}\,2\alpha}{1+\text{Cos}\,2\alpha + \text{Sin}\,2\alpha} = \tan\alpha$

m) $1 - \text{Sin}4\alpha + \text{Cotg}\left(\frac{3\pi}{4} - 2\alpha\right) \,.\, \text{Cos}4\alpha = 0$

n) $\frac{\sqrt{2} - \text{Sin}\,\alpha - \text{Cos}\,\alpha}{\text{Sin}\,\alpha - \text{Cos}\,\alpha} = \tan\left(\frac{\alpha}{2} - \frac{\pi}{8}\right)$

2. Aşağıdakileri sadeleştiriniz.

a) $\frac{2\,(\text{Sin}\,2\alpha + 2\,\text{Cos}^2\alpha - 1)}{\text{Cos}\,\alpha - \text{Sin}\,\alpha - \text{Cos}3\,\alpha + \text{Sin}3\,\alpha}$

b) $\frac{1+\text{Sin}\,2x}{(\text{Sin}\,x + \text{Cos}\,x)^2}$

c) $1 + \text{Cos}\frac{\pi}{7} + \text{Cos}\frac{2\pi}{7} + \text{Cos}\frac{3\pi}{7} + \text{Cos}\frac{4\pi}{7} + \text{Cos}\frac{5\pi}{7} + \text{Co}$

d) $\frac{\text{Cot}\,4\alpha + 1}{\text{Cotg}\,\alpha - \tan\alpha}$

e) $\frac{\text{Cos}\,2\alpha}{\text{Sin}^2\,2\alpha\,(\text{Cotg}^2\,\alpha - \tan^2\alpha)}$

f) $\frac{48 \,.\, \text{Sin}80 \,.\, \text{Sin}65 \,.\, \text{Sin}35}{\text{Sin}20 + \text{Sin}50 + \text{Sin}110}$

g) $8\,(\text{Cos}\,24 + \text{Cos}\,48 - \text{Cos}\,84 - \text{Cos}\,12)$

h) $256 \,.\, \text{Sin}^2 20 \,.\, \text{Sin}^2 20 \,.\, \text{Sin}^2 60 \,.\, \text{Sin}^2 80$

i) $\tan 9 - \tan 63 + \tan 81 - \tan 27$

j) $\frac{\text{Cotg}\,15 + 1}{2\,\text{Cotg}\,15}$

k) $\text{Cot}\frac{\pi}{7} \,.\, \text{Cos}\frac{4\pi}{7} \,.\, \text{Cos}\frac{5\pi}{7} = ? \quad \left(\frac{1}{8}\right)$

l) $\text{Arc Sin}\left(-\frac{\sqrt{2}}{2}\right) + \text{Arc Cos}\left(-\frac{1}{2}\right) - \text{Arc tan}\left(-\sqrt{3}\right)$

m) $\text{Arc Cos}\left(\text{Cos}\left(-\frac{\sqrt{3}}{2}\right)\right)$

n) $\text{Arc tan}\sqrt{2} + \text{Arc tan}\frac{1}{\sqrt{2}}$

o) $\text{Cos}\left(2\,\text{Arc Sin}\frac{2}{5}\right)$

p) $\text{Sin}\left(\frac{1}{2}\,\text{Arc Cos}\frac{1}{9}\right)$

r) $\text{Sin}\left(\text{Arc Sin}\frac{3}{5} - \text{Arc Cos}\frac{3}{5}\right)$

s) $\text{Cos}\left(\text{Arc Sin}\frac{4}{5}\right)$

t) $\text{Sin}\,(2\,\text{Arc tan}\,2)$

3. Aşağıdaki denklemleri çözünüz.

a) $\tan(2x+3)=\sqrt{3}$

b) $\sin(5x-2)=-1$

c) $\sqrt{2}\cdot\cos^2 7x-\cos 7x=0$

d) $2\sin x+\tan x=0$

4. Aşağıdaki eşitsizlikleri çözünüz.

a) $\sin x \leq \frac{1}{2}$ b) $\sin x \geq \frac{\sqrt{3}}{2}$

c) $\cos x \leq -\frac{1}{2}$ d) $\sin x < \cos x$

e) $\sin 3x < \sin x$

büyük
yayınları

YENİ SINAV SİSTEMİNE GÖRE HAZIRLANMIŞTIR

ÖSS + ÖYS

için

TÜRKÇE

404 TEST SORULU

Erdoğan Turhan